

## Parallelograms in *Glencoe Geometry*

Below, I provide a summary of the mathematics leading up to the treatment of parallelograms presented in the 9th/10th grade geometry textbook currently used in East Baton Rouge Parish, *Geometry: Integration, Applications, and Connections*, Glencoe 1998.

**I. Vocabulary:** *Point, line, angle, supplement of an angle, vertical angles, perpendicular lines, parallel lines, triangle, quadrilateral, parallelogram, rhombus, rectangle, square, polygon, congruent segments, congruent angles, congruent triangles.*

**II. Constructions.** *Glencoe Geometry* demonstrates the following ruler-and-compass constructions:

- A. Given a segment, construct its perpendicular bisector.
- B. Given an angle, construct the bisector of that angle.
- C. Given a line and a point, construct the line through the point perpendicular to the given line.

The constructions guarantee the *existence* of lines with properties as stipulated. Introducing such lines is often an important step in a demonstration.

**III. Facts presented but not proved in *Glencoe Geometry*.** These include some “axioms,” or unproved assertions (*e.g.*, SAS and the parallel postulate) of classical systems of geometry such as Euclid as well as some derived truths, or “theorems” (*e.g.*, ASA, SSS and **D**).

- A. Supplements of congruent angles are congruent; vertical angles are congruent.
- B. SAS, ASA and SSS criteria for triangle congruence.
- C. *The parallel postulate.* If parallel lines are cut by a transversal, then corresponding angles are congruent.
- D. If two lines are both perpendicular to a third line, then the two lines are parallel.

The facts listed above do not include every fundamental truth that one might use in a geometric argument. For example, “Two points determine a line,” is certainly a fact that we appeal to over and over again, but it is so well-known that I omitted it from the list, assuming that no one needs to be reminded. *However*, if one sets out to sort and organize truths of geometry with utmost care, then one would certainly try to include explicit statements such “obvious” facts as well as the more profound things such as SAS.

**IV. Theorems.** Each of the following is proved in *Glencoe Geometry*, or the proof is assigned as an exercise.

- A. Given a line and a point not on it, there is a line through that point that is parallel to the given line.
- B. The sum of the measures of the angles of a triangle is  $180^\circ$ .
- C. If two lines are cut by a transversal and corresponding angles are congruent, then the two lines are parallel.
- D. *Parallelogram theorem.* For any quadrilateral, the following are equivalent:
  - i.* Opposite sides are parallel;
  - ii.* The diagonals bisect one another;
  - iii.* Opposite sides are congruent;
  - iv.* Opposite angles are congruent.
- E. *Rhombus theorem:* The diagonals of a parallelogram are perpendicular if and only if all four sides are congruent. *Rectangle theorem:* The diagonals of a parallelogram are congruent if and only if all adjacent sides are perpendicular. *Note:* there is a quadrilateral that is neither a rhombus nor a rectangle, but whose diagonals are both perpendicular and congruent to one another.