Student Project

The triangle area reduced to parallelogram area by cutting up triangles

Goal: Students will 1) demonstrate how to cut triangle by straight line parallel to the base and reassemble the pieces to form a parallelogram, 2) determine what mathematical assumptions are necessary to justify the construction and 3) prove that the construction always does what it is intended.

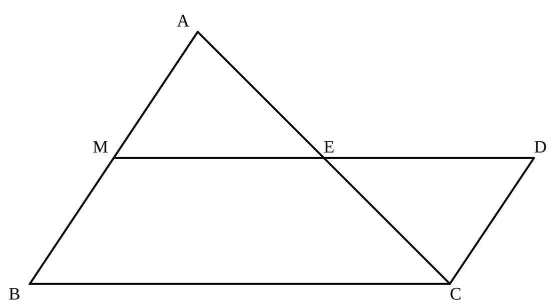
Part I. The purpose of this part is to develop a visual understanding of the mathematical ideas.

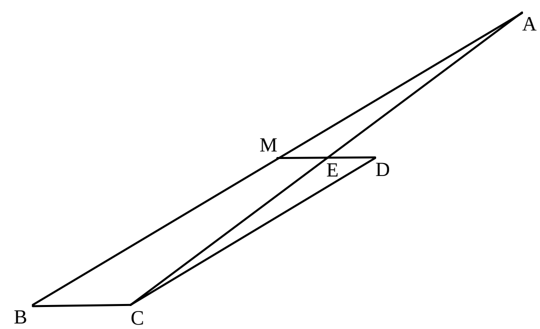
A. Illustrate how, in a right triangle, the line joining the midpoint of the hypotenuse to the midpoint of a leg cuts the triangle into two pieces that fit together to form a rectangle.

B. Illustrate how, in any triangle, the line joining the midpoints of any two sides cuts the triangle into two pieces that fit together to form a parallelogram.

Part II. Prove the following:

Lemma A. Let ABC be a triangle and let M be the midpoint of AB. Let m be the line through M parallel to BC and let l be the line through C parallel to AB. Let D be the point where l and m meet and let E be the point where m crosses AC. Then ECD is congruent to EAM.





(Hint: Because $MBCD$ is a parallelogram, MB is congruent to DC.)

Lemma B. Let ABC be a triangle and let M be the midpoint of AB. Let MP be the perpendicular from M to the line containing BC. Let AQ be the perpendicular from A to the line containing BC, and let N be the midpoint of AQ. Then NQ is congruent to MP.

Theorem. Given any triangle with one side identified as base, that triangle has the same area as a parallelogram on the same base and of height half the height of the given triangle.

Part III. Where and how is the Parallel Postulate used in Part II? (Note that it is NOT needed to construct the parallel to BC through M.)

Part IV. One observes similar triangles in the figures drawn above. Are any facts from the theory of similarity (Euclid Book VI) indispensable for carrying out the arguments in Part II?