

19. Under H_0 , a random variable has the cumulative distribution function $F_0(x) = x^2$, $0 \leq x \leq 1$; and under H_1 , it has the cumulative distribution function $F_1(x) = x^3$, $0 \leq x \leq 1$.
- If the two hypotheses have equal prior probability, for what values of x is the posterior probability of H_0 greater than that of H_1 ?
 - What is the form of the likelihood ratio test of H_0 versus H_1 ?
 - What is the rejection region of a level α test?
 - What is the power of the test?
20. Consider two probability density functions on $[0, 1]$: $f_0(x) = 1$, and $f_1(x) = 2x$. Among all tests of the null hypothesis $H_0 : X \sim f_0(x)$ versus the alternative $X \sim f_1(x)$, with significance level $\alpha = 0.10$, how large can the power possibly be?
21. Suppose that a single observation X is taken from a uniform density on $[0, \theta]$, and consider testing $H_0 : \theta = 1$ versus $H_1 : \theta = 2$.
- Find a test that has significance level $\alpha = 0$. What is its power?
 - For $0 < \alpha < 1$, consider the test that rejects when $X \in [0, \alpha]$. What is its significance level and power?
 - What is the significance level and power of the test that rejects when $X \in [1 - \alpha, 1]$?
 - Find another test that has the same significance level and power as the previous one.
 - Does the likelihood ratio test determine a unique rejection region?
 - What happens if the null and alternative hypotheses are interchanged— $H_0 : \theta = 2$ versus $H_1 : \theta = 1$?
22. In Example A of Section 8.5.3 a confidence interval for the variance of a normal distribution was derived. Use Theorem B of Section 9.3 to derive an acceptance region for testing the hypothesis $H_0: \sigma^2 = \sigma_0^2$ at the significance level α based on a sample X_1, X_2, \dots, X_n . Precisely describe the rejection region if $\sigma_0 = 1$, $n = 15$, $\alpha = .05$.
23. Suppose that a 99% confidence interval for the mean μ of a normal distribution is found to be $(-2.0, 3.0)$. Would a test of $H_0: \mu = -3$ versus $H_A: \mu \neq -3$ be rejected at the .01 significance level?
24. Let X be a binomial random variable with n trials and probability p of success.
- What is the generalized likelihood ratio for testing $H_0: p = .5$ versus $H_A: p \neq .5$?
 - Show that the test rejects for large values of $|X - n/2|$.
 - Using the null distribution of X , show how the significance level corresponding to a rejection region $|X - n/2| > k$ can be determined.
 - If $n = 10$ and $k = 2$, what is the significance level of the test?
 - Use the normal approximation to the binomial distribution to find the significance level if $n = 100$ and $k = 10$.