(*Hint*: In order for Y to equal k, how many successes must result in the first $k-1$ trials and what must be the outcome of trial k?)

(d) Show that

$$E[Y] = r/p$$

(*Hint*: Write $Y = Y_1 + \cdots + Y_r$ where Y_i is the number of trials needed to go from a total of $i-1$ to a total of i successes.)

21. If U is uniformly distributed on $(0, 1)$, show that $a + (b-a)U$ is uniform on (a, b).

22. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 10 minutes? If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

23. If X is a normal random variable with parameters $\mu = 10, \sigma^2 = 36$, compute

(a) $P\{X > 5\}$;
(b) $P\{4 < X < 16\}$;
(c) $P\{X < 8\}$;
(d) $P\{X < 20\}$;
(e) $P\{X > 16\}$.

24. The Scholastic Aptitude Test mathematics test scores across the population of high school seniors follow a normal distribution with mean 500 and standard deviation 100. If five seniors are randomly chosen, find the probability that **(a)** all scored below 600 and **(b)** exactly three of them scored above 640.

25. The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 40, \sigma = 4$. What is the probability that in 2 of the next 4 years the rainfall will exceed 50 inches? Assume that the rainfalls in different years are independent.

26. The weekly demand for a product approximately has a normal distribution with mean 1,000 and standard deviation 200. The current on hand inventory is 2,200 and no deliveries will be occurring in the next two weeks. Assuming that the demands in different weeks are independent,

(a) what is the probability that the demand in each of the next 2 weeks is less than 1,100?

(b) what is the probability that the total of the demands in the next 2 weeks exceeds 2,200?

27. A certain type of lightbulb has an output that is normally distributed with mean 2,000 end foot candles and standard deviation 85 end foot candles. Determine a lower specification limit L so that only 5 percent of the lightbulbs produced will be defective. (That is, determine L so that $P\{X \geq L\} = .95$, where X is the output of a bulb.)

28. A manufacturer produces bolts that are specified to be between 1.19 and 1.21 inches in diameter. If its production process results in a bolt's diameter being normally distributed with mean 1.20 inches and standard deviation .005, what percentage of bolts will not meet specifications?

29. Let $I = \int_{-\infty}^{\infty} e^{-x^2/2}\,dx$.

(a) Show that for any μ and σ

$$\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2}\,dx = 1$$

is equivalent to $I = \sqrt{2\pi}$.

(b) Show that $I = \sqrt{2\pi}$ by writing

$$I^2 = \int_{-\infty}^{\infty} e^{-x^2/2}\,dx \int_{-\infty}^{\infty} e^{-y^2/2}\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy$$

and then evaluating the double integral by means of a change of variables to polar coordinates. (That is, let $x = r\cos\theta, y = r\sin\theta, dx\,dy = r\,dr\,d\theta$.)

30. A random variable X is said to have a lognormal distribution if $\log X$ is normally distributed. If X is lognormal with $E[\log X] = \mu$ and $\text{Var}(\log X) = \sigma^2$, determine the distribution function of X. That is, what is $P\{X \leq x\}$?

31. The lifetimes of interactive computer chips produced by a certain semiconductor manufacturer are normally distributed having mean 4.4×10^6 hours with a standard deviation of 3×10^5 hours. If a mainframe manufacturer requires that at least 90 percent of the chips from a large batch will have lifetimes of at least 4.0×10^6 hours, should he contract with the semiconductor firm?

32. In Problem 31, what is the probability that a batch of 100 chips will contain at least 4 whose lifetimes are less than 3.8×10^6 hours?

33. The lifetime of a television picture tube is a normal random variable with mean 8.2 years and standard deviation 1.4 years. What percentage of such tubes lasts

(a) more than 10 years;
(b) less than 5 years;
(c) between 5 and 10 years?

34. The annual rainfall in Cincinnati is normally distributed with mean 40.14 inches and standard deviation 8.7 inches.

(a) What is the probability this year's rainfall will exceed 42 inches?
(b) What is the probability that the sum of the next 2 years' rainfall will exceed 84 inches?
(c) What is the probability that the sum of the next 3 years' rainfall will exceed 126 inches?
(d) For parts (b) and (c), what independence assumptions are you making?

35. The height of adult women in the United States is normally distributed with mean 64.5 inches and standard deviation 2.4 inches. Find the probability that a randomly chosen woman is

(a) less than 63 inches tall;
(b) less than 70 inches tall;
(c) between 63 and 70 inches tall.
(d) Alice is 72 inches tall. What percentage of women is shorter than Alice?
(e) Find the probability that the average of the heights of two randomly chosen women exceeds 66 inches.
(f) Repeat part (e) for four randomly chosen women.

36. An IQ test produces scores that are normally distributed with mean value 100 and standard deviation 14.2. The top 1 percent of all scores are in what range?

37. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 1$.

(a) What is the probability that a repair time exceeds 2 hours?
(b) What is the conditional probability that a repair takes at least 3 hours, given that its duration exceeds 2 hours?

38. The number of years a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Jones buys a used radio, what is the probability that it will be working after an additional 10 years?

39. Jones figures that the total number of thousands of miles that a used auto can be driven before it would need to be junked is an exponential random variable with parameter $\frac{1}{20}$. Smith has a used car that he claims has been driven only 10,000 miles. If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it? Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed but rather is (in thousands of miles) uniformly distributed over (0, 40).

***40.** Let $X_1, X_2, \ldots, X_n$ denote the first n interarrival times of a Poisson process and set $S_n = \sum_{i=1}^{n} X_i$.

(a) What is the interpretation of S_n?
(b) Argue that the two events $\{S_n \leq t\}$ and $\{N(t) \geq n\}$ are identical.
(c) Use part (b) to show that

$$P\{S_n \leq t\} = 1 - \sum_{j=0}^{n-1} e^{-\lambda t}(\lambda t)^j/j!$$

(d) By differentiating the distribution function of S_n given in part (c), conclude that S_n is a gamma random variable with parameters n and λ. (This result also follows from Corollary 5.7.2.)

***41.** Earthquakes occur in a given region in accordance with a Poisson process with rate 5 per year.

(a) What is the probability there will be at least two earthquakes in the first half of 2010?
(b) Assuming that the event in part (a) occurs, what is the probability that there will be no earthquakes during the first 9 months of 2011?
(c) Assuming that the event in part (a) occurs, what is the probability that there will be at least four earthquakes over the first 9 months of the year 2010?

***42.** When shooting at a target in a two-dimensional plane, suppose that the horizontal miss distance is normally distributed with mean 0 and variance 4 and is independent of the vertical miss distance, which is also normally distributed with mean 0 and variance 4. Let D denote the distance between the point at which the shot lands and the target. Find $E[D]$.

43. If X is a chi-square random variable with 6 degrees of freedom, find

(a) $P\{X \leq 6\}$;
(b) $P\{3 \leq X \leq 9\}$.

44. If X and Y are independent chi-square random variables with 3 and 6 degrees of freedom, respectively, determine the probability that $X + Y$ will exceed 10.

45. Show that $\Gamma(1/2) = \sqrt{\pi}$ (*Hint*: Evaluate $\int_0^\infty e^{-x}x^{-1/2}\,dx$ by letting $x = y^2/2$, $dx = y\,dy$.)

46. If T has a t-distribution with 8 degrees of freedom, find **(a)** $P\{T \geq 1\}$, **(b)** $P\{T \leq 2\}$, and **(c)** $P\{-1 < T < 1\}$.

47. If T_n has a t-distribution with n degrees of freedom, show that T_n^2 has an F-distribution with 1 and n degrees of freedom.

48. Let Φ be the standard normal distribution function. If, for constants a and $b > 0$

$$P\{X \leq x\} = \Phi\left(\frac{x-a}{b}\right)$$

characterize the distribution of X.