M4056 Quiz 5, Sept. 24, 2010 Name________________________

1. Recall that the Poisson distribution is a discrete distribution with *pmf*

$$f(x|\lambda) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

   a. Let $\vec{X} = (X_1, \ldots, X_n)$ be a sample from a Poisson distribution. Write the sample *pmf* explicitly as a function of $\vec{x} = (x_1, \ldots, x_n)$:

$$f_{\vec{X}}(\vec{x}|\lambda) =$$

   b. Show that $\sum_{i=1}^{n} X_i$ is a sufficient statistic for λ.

2. Suppose that $X_1, \ldots, X_n$ are independent and uniformly distributed on the open interval on the real line between α and β. Then

$$f_{X_i}(x_i \mid (\alpha, \beta)) = \begin{cases} \frac{1}{\beta - \alpha}, & \text{if } \alpha < x_i < \beta; \\ 0, & \text{otherwise.} \end{cases}$$

   Show that $T(\vec{X}) = (\min\{X_1, \ldots, X_n\}, \max\{X_1, \ldots, X_n\})$ is a sufficient statistic for the vector parameter (α, β).