M4056 Quiz, Aug. 28, 2010 Name________________________

Recall: If X is binomial (n,p), then $f_X(x \mid n,p) = \binom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, \ldots, n$.

1.a) Assume X is binomial (n,p). What is the probability that $a \le X \le b$ for integers a, b between 0 and n (inclusive)?

$$P(a \le X \le b) = \sum_{x=a}^{b} \binom{n}{x} p^x (1-p)^{n-x}$$

1.b) Assume X is binomial $(5, 1/3)$. What is the probability that $2 \le X \le 4$?

$$\binom{5}{2} \tfrac{1}{3}^2 \tfrac{2}{3}^3 + \binom{5}{3} \tfrac{1}{3}^3 \tfrac{2}{3}^2 + \binom{5}{4} \tfrac{1}{3}^4 \tfrac{2}{3}$$

$$= [10 \cdot 8 + 10 \cdot 4 + 5 \cdot 2]/243 = 125/243$$

1.c) Write the expression that gives the *mgf* of X (i.e., the expected value of e^{tX}). (I am not asking you to simplify.)

$$E(e^{tX}) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x (1-p)^{n-x}$$

2) Assume X is binomial (n,p). Define $g(p) := f_X(x \mid n,p)$. Compute $\frac{d}{dp} g(p)$, and use this to find the maximum of g on $[0,1]$.

$$g'(p) = \frac{d}{dp} \binom{n}{x} p^x (1-p)^{n-x} = \binom{n}{x} \left[x p^{x-1} (1-p)^{n-x} + p^x (n-x)(1-p)^{n-x-1} (-1) \right]$$

At a maximum, $g'(p)$ vanishes, so

$$0 = x p^{x-1} (1-p)^{n-x} - p^x (n-x)(1-p)^{n-x-1}$$

$$= p^{x-1} (1-p)^{n-x-1} \left(x(1-p) - p(n-x) \right).$$

So $x - px = pn - px$

so $x = pn$

$$\boxed{\text{so} \quad p = \frac{x}{n}.}$$

This is the value of p at which the maximum occurs. The value of g at the max is: $\binom{n}{x} \left(\frac{x}{n}\right)^x \left(\frac{n-x}{n}\right)^{n-x}$