Notes on Lecture 2

James J. Madden

time	Topic and comments by JJM
1:20	Basic general definition from the point of view of sets. **Definition.** f:A→B means that f is a rule which assigns to each element of set A an element of set B.
2:00	Pictorial representation of function f, its domain, A, and range, B.
2:30	The meaning of "each"; it's role in making a function "well-defined"
3:15	Notice: B does not have to be used up. Notice: Different elements of A might be assigned to the same element of B.
4:00	A is called the domain of f. A = dom f B is called the range of f. (Note. Today many people use the word "codomain" for what these lectures call "range".)
5:10	The image of f is the part of the range that "used up." The image of f is { f(a) : a in A }. (Today, some people use the term "range" to mean what Herb calls the image. This can be confusing, unless we are always willing to explain what we mean. A mathematician is a person who is always ready to explain what she means.)
6:45	"f is onto" means: the image of f equals the range of f. (Today many people use the word "surjective" for what these lectures call "onto".)
7:15	Example: If A={1,2,3}, B={4,8,12} and f(x)=4x, then f:A→B is "onto"
9:52 to 12:50	"f:A→B is one-to-one" means: different elements of the domain map to different elements of the range, i.e., no two elements of the domain have the same image. (Today we also use the word "injective" to describe a function with this property.)
12:56	A function that is both one-to-one and onto has an inverse. We can picture the inverse by "reversing the arrowheads." Herb says, "If the function is **not** one-to-one and onto, we cannot reverse the arrowheads--believe it or not!"
14:36	Wrap Up
14:50	Back to $s = 16t^2$
15:40	The discussion here shows why it is natural (and necessary) in applications to keep the domain in mind.
17:50	Intervals are important domains in real-life laboratory situations.
18:30	Intervals (open and closed)
20:37	Pictures and notation
21:33	Neighborhoods. A neighborhood of c is any **open** interval that contains c.
23:10	If h>0, then (c-h, c+h) is a symmetric neighborhood of c.
24:20	If a neighborhood of c is not symmetric, there is a smaller neighborhood that is symmetric.
24:48	"Deleted neighborhoods" include points close to c, but not c itself. We use these when we are interested in what happens NEAR c but do not care about what happens AT c.
26:25	What does "near" mean? We need the concept of distance to say.
27:00	Distance and absolute value. |x-3| is the distance from 3 to x.
29:00	A unnecessarily laborious way of saying what absolute value means.
30:20	f = g means: f and g have the same domain, and f(x) = g(x) for all x in the domain.

	f+g is the function whose domain is the intersection of dom f and dom g, and whose rule is: given input x, find f(x) and g(x) and then add these numbers together.
34:00	Composition of functions. If f and g are functions, then the composition of f and g --- denoted $g \circ f$, and read "g after f"--- is the function that takes an input, applies f and then applies g to the output of f .

Discussion Problems

1) At the beginning of the lecture, Herb comments that this lecture is about functions, and he says that a function "is a relation between variables." However, the definition that Herb gives a few moments later does not mention variables. His first statement is (choose one):
   a) a different yet complete and accurate way of describing functions;
   b) the way that functions are defined in high school, but not college;
   c) an off-the-cuff description of functions that is perfectly acceptable;
   d) potentially misleading, but not incorrect;
   e) wrong;
   f) meaningless;
   g) none of the above.

2) Explain what a function is.

3) Using a pictorial representation similar to the one used at the beginning (with disks/blobs to represent sets, dots to represent elements and arrows to depict the rule of the function), illustrate the meaning of the composition of two functions.

4) True or false: In order for $g \circ f$ to make any sense, the image of f must be contained in the domain of g. Justify your answer.