## Lesson 5: One-Variable Inequalities

*Objectives*
- The students will be able to solve one-variable inequalities.
- The students will be able to graph and describe the solution to one-variable inequalities.
- The students will be able to solve one-variable inequalities related to real-world applications.

*Essential Questions*
- How are solutions to inequalities described in written and graphed formats?

*Tools*
- Student Journal
- Setting the Stage transparency
- Dry–erase boards, markers, erasers
- Overhead projector
- Overhead Tiles
- Student Tiles
- Tile Pad
- Inequality Tile Pad
- Inequality Tile Pad Transparency
- Poster Paper and Markers

*Warm Up*
- Problems of the Day

*Number of Days*
- 2 days (A suggestion is to complete activities 1 and 2 with Practice Exercises 1 through 4 on the first day. On the second day, have students complete Activity 3, the remaining Practice Exercises, and Lesson Quiz.)

*Vocabulary*

OR
Inequality
Less than or equal to
Greater than or equal to
Compound inequality
Subtraction Property of Inequality
Division Property of Inequality
ADD
Less than
Greater than
Compound
Addition Property of Inequality
Multiplication Property of Inequality



**Teacher Reference**

**Setting the Stage**
The goal of this Setting the Stage is to help students review and develop their intuitive understanding of inequalities from real-world examples. After this Setting the Stage, you can transition students to the formal term "inequality", and the symbolic method and number line method to represent inequalities. Place the transparency on the overhead projector and then ask the students if they know any of the requirements, ratings, or conditions to these real-world situations. You may need to supply the students with some of the following information orally or by writing it on the transparency:

**Movie Ratings:** The MPAA (Motion Picture Association of America) movie ratings were:

- **G-General Audiences**: All ages admitted.
- **PG - Parental Guidance Suggested**: Some material may not be suitable for children.
- **PG-13 - Parents Strongly Cautioned**: Some material may be inappropriate for children under 13.
- **R – Restricted**: Under 17 requires accompanying parent or adult guardian.
- **NC-17:** No One 17 and Under Admitted.

**FBI Age Requirements:** Many federal investigative jobs have age requirements, and applicants must be at least 21 years of age and under the age of 37 at the time of appointment.

**Space Shuttle Launch Conditions:** The countdown for a Shuttle launch will stop if the temperature exceeds 99 degrees for more than 30 consecutive minutes. And in no case may the Space Shuttle launch if the temperature is 35 degrees or colder.

Now, have the class work in pairs on their dry-erase boards. Each pair needs to pick at least one condition and represent it as many ways as possible. Encourage students to use drawings, words, symbols, and graphs to represent the conditions. You may want to allow students to pick another real-world application of their choice that has inequality characteristics. Other possible applications for inequalities that you may want to use are age requirements for various reality TV shows, military age requirements, college requirements for standardized test scores, rental car age requirements, and driver license age requirements. Walk around to look at how students represent each condition or have the pairs hold up their dry-erase boards to show you and the class. Don't worry if students do not formally use number lines or inequality statements correctly at this point. Mainly, you are helping students link to prior understanding and to share their current thinking about inequalities. You will have a chance, if needed, to formally introduce students to graphing and representing inequalities as you transition to the first activity.

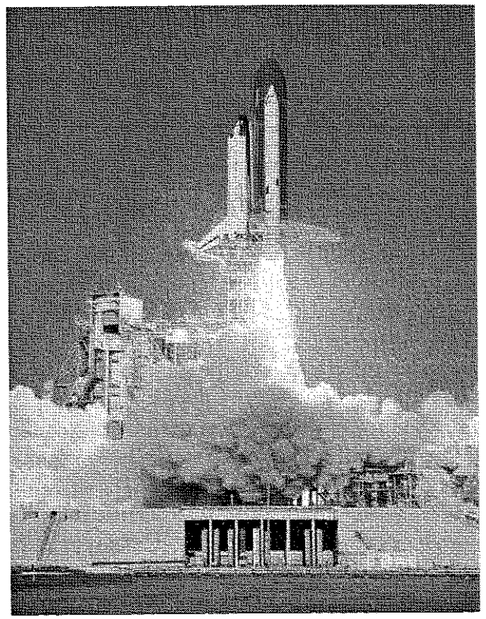
Have the students keep the work they did on their dry-erase boards, they will get a chance to work with their representations again in the activity.

**Option:** Put a number line at the front of class and have students stand at different locations to represent various inequalities. For example, you could have all the students stand in such a way that they all need to be in a position on the number line that represents $x < 3$.

**Note:** You may want to let students know that there are three possible relationships between any two real numbers (=, <, >). Examples: $5 = 5$, $5 < 6$, and $5 > 4$.

**Setting the Stage** **Transparency 1**


# Movie Ratings



# Space Shuttle Launch Conditions


# FBI Age Requirements



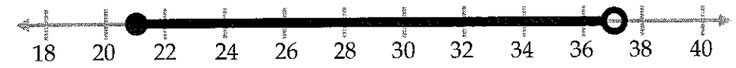
**Teacher Reference**

**Activity 1**

The goal of this activity is to introduce the students to the more formal methods to represent inequalities, with words, graphs (number lines), and symbols (inequalities).

Display the Setting the Stage transparency. Use one of the conditions from the Setting the Stage and guide the class through representing the condition with the formal words, symbols, and number line. For example, you may want to lead the students through representing the age requirements for the FBI.

- A person applying for the FBI must be 21 or older and less than 37 years of age.
- Using "$a$" to represent age, a mathematical inequality would be $21 \leq a < 37$.
- A number line representation could be:




Model your thinking aloud or solicit help from students. You may want to use terms such as *endpoints, range, interval, variable, less than, greater than, less than or equal to,* and *greater than or equal to* as you think out loud. You will also want to briefly describe when to fill in the endpoints or leave them hollow.

Now have pairs of students go back to the condition they chose for the Setting the Stage and make any appropriate changes on the dry-erase boards to represent their condition formally with words, symbols (inequality), and a graph (number line). You can walk around the classroom to assess the students and have students share with the class as needed. You may want to ask guiding questions, such as:

- How do the specific characteristics on your graph reflect the real-world concept?
- What advantage does the graph have compared to the real world scenario?
- What advantage does the symbol representation have compared to the graph?

For Exercise 1, have the students record the representations for their condition.

Now, write the term *inequality* in the center of the board and have the students create a graphic organizer or word web for the term. You can have various students come to the front to add to the organizer.

Continue to have student pairs work on Exercise 2. Have student volunteers share their results with the class. If no student gave a fraction as an answer, ask the class, "Can we also place the $w$ at 3/2?" Place $w$ anywhere on the transparency number line so that $w$ is less than 3. Discuss with the class that $w$ can have many possible values when it is described as *a number less than 3*.

Have the student pairs complete Exercise 3 and then share their results with the class. Then have the class complete Exercises 4 through 6 and share their results. After the class has shared results, have them continue with Exercises 7 through 12. While the class is working on the exercises, walk around checking the students understanding of the material and answer questions they may have. If a student pair has any intriguing questions, pose the question to the rest of the class and let other student pairs give answers. After the class has completed the activity, have student volunteers share their results on Exercises 7 through 12. Lead a follow-up discussion with the class on what they have learned about graphing inequalities and writing symbolic and descriptive inequalities. Have a student volunteer list the class responses on the board or on a blank transparency on the overhead projector. Tell the class to record information they find useful.

**SJ Page 57**
**Activity 1**

**1.** Record the different representations for the condition you chose.

*Answers will vary.*

**2.** Write the inequality that represents $w$ less than 3. ***The inequality is $w < 3$.***

**a.** With your partner draw a number line, then each of you place a $w$ anywhere on the number line, so that $w$ is less than 3. You cannot choose the same place on the number line. ***Answers will vary. See number lines as needed.***

**b.** Write the placement location of the $w$ on your number line.
***Answers will vary. A sample response might be –2.***

**c.** Write your partner's placement location. ***Answers will vary. A sample response might be 1.***

**d.** Whose value, or location, represents the largest number? Explain why. Now, write an inequality which represents a true statement for all three values, including the 3.
***Answers will vary. A sample response might be: "My partner placed w at 1. My partner had the larger number because 1 is greater than –2." The inequality for all three values is $-2 < 1 < 3$.***

**e.** Write a non–integer value which is greater than your $w$ value, but less than 3.
***Answers will vary. A sample response might be: –1/2.***

**3.** Erase the variables and then place other variables on your number line according to the conditions (or criteria) below and write the corresponding inequalities.

**a.** $w$ is greater than or equal to –2 $\quad w \geq -2$

**b.** $x$ is between –3 and 2, excluding the endpoints –3 and 2 $\quad -3 < x < 2$

**c.** $y$ is more than 0 but less than or equal to 5 $\quad 0 < y \leq 5$

**d.** $z$ is greater than 5 or less than –4 $\quad z > 5$ *or* $z < -4$
***Note: This statement cannot be represented with one statement like in part c. You may want to briefly discuss with students the reason.***



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**4.** Erase the variables and consider the following new criteria for the variables.

| | |
|---|---|
| $w$ is between 0 and 1 | $x$ is between –1 and 0 |
| $y$ is between 4 and 5 | $z$ is between –3 and –2 |

**a.** Determine if $w < x$ is always true, sometimes true, or never true. Give examples with numbers to help determine the answer.
***Sample response: Never true. For example 0.5 is not less than –0.5.***

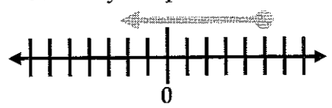
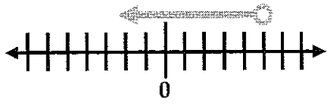
**b.** Determine if $wy > xy$ is always true, sometimes true, or never true.
***Sample response: Always true. For example, 0.5(4.5) is 2.25 which is greater than –0.5(4.5) which is –2.25.***

**5.** **a.** Write two inequality questions for the variables $w$, $x$, $y$, and $z$ from Exercise 4. A sample question could be, "Is $w < x$ always true, sometimes true, or never true?"
***Answers will vary. A sample response might be: "Is $y > x$ always, sometimes, or never true?" and "Is $-1(z) > w$?"***

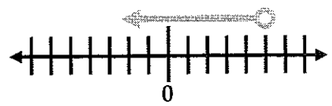
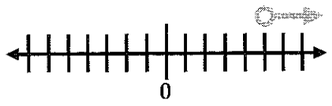
**b.** Trade questions with your partner and answer your partner's inequality questions.
***Answers will vary.***

**6.** Discuss your inequality questions with another student pair. How were your inequality questions similar and how were they different?
***Answers will vary.***

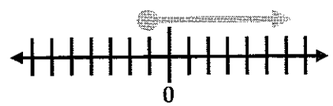
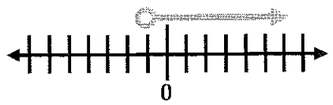
**7.** Below are several pairs of correctly and incorrectly graphed inequalities. For each incorrect graph describe what is incorrect.

| | | Correctly Graphed | Incorrectly Graphed |
|---|---|---|---|
| **a.** | $x \leq 5$ |  |  |

***Sample response: The endpoint should be solid.***

| | | | |
|---|---|---|---|
| **b.** | $x < 5$ |  |  |

***Sample response: The arrow is pointed in the wrong direction.***

| | | | |
|---|---|---|---|
| **c.** | $-1 \leq x$ |  |  |

***Sample response: The endpoint should be solid.***

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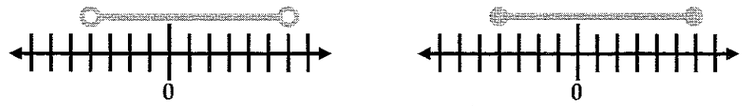
**d.** $-1 < x$

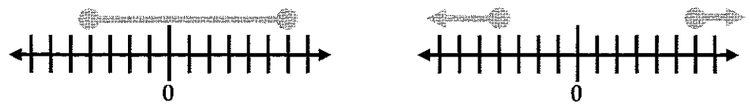

***Sample response: The arrow is pointed in the wrong direction.***

**e.** $-4 < x < 6$




***Sample response: The endpoints should not be solid.***

**f.** $-4 \leq x \leq 6$




***Sample response: There should be no arrows and the graph should have the numbers between –4 and 6 not outside that range.***

**8.** What are the characteristics of a correctly graphed inequality?
***Answers will vary. A sample response might be: "The endpoints are solid if the inequality has less than or equal to."***

**9.** Write a description for each correctly graphed inequality from Exercise 7. The first one has been completed for you.

**a.** $x \leq 5$: All real numbers less than or equal to a positive five.

**b.** $x < 5$: ***Sample response: x is all real numbers less than five.***

**c.** $-1 \leq x$: ***Sample response: x is all real numbers greater than or equal to negative one.***

**d.** $-1 < x$: ***Sample response: x is all real numbers greater than negative one.***

**e.** $-4 < x < 6$: ***Sample response: x is all real numbers between negative four and positive six.***

**f.** $-4 \leq x \leq 6$: ***Sample response: x is all real numbers greater than or equal to negative four and less than or equal to six.***



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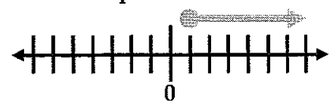
You will often see the variable of an inequality on the left, such as the $x$ in the inequality $x \geq 1$. A written or verbal description of this inequality would generally be "$x$ is greater than one." There are, however, other equivalent methods to represent this inequality with symbols. There are also other methods to write or verbally describe this inequality. Below are some examples. The graph of the inequality remains the same for the different symbols, written, or verbal representations.

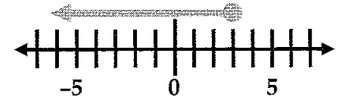
**Symbol methods:** $x \geq 1$, $1 \leq x$
**Written or verbal:** $x$ is greater than or equal to 1,
1 is less than or equal to $x$

**Graph:**



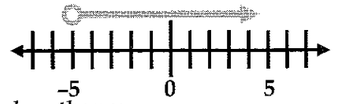

**10.** For each written description below of an inequality, draw a graph of the inequality, then write two different symbol inequalities and another written or verbal description.

**a.** $w$ is less than or equal to positive three.



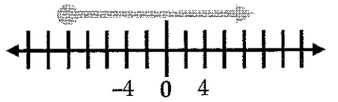

Symbols: $w \leq 3$, $3 \geq w$
Written or verbal: ***Sample response: Positive three is greater than or equal to w.***

**b.** $y$ is greater than negative five.



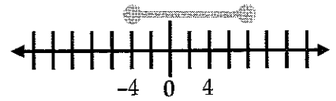

Symbols: $y > -5$, $-5 < y$
Written or verbal: ***Sample response: Negative five is less than y.***

**c.** An unknown number is at least a negative ten.




Symbols: $x \geq -10$, $-10 \leq x$
Written or verbal: ***Sample response: Negative ten is less than or equal to some number.***

**d.** An unknown number is at least negative four but no more than positive eight.




Symbols: $-4 \leq x \leq 8$, $8 \geq x \geq -4$

Written or verbal: ***Sample response: An unknown number is no more than positive eight but at least negative four.***

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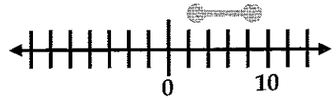
**11.** Write a symbolic inequality for each real-world example, then draw a graph.

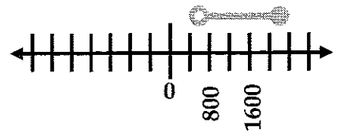
**a.** Male adult Nile crocodiles usually mature around 10 years with an approximate length of 3 meters. One of the largest recorded adult male Nile crocodiles measured 8.6 meters.

$3 \le l \le 8.6$



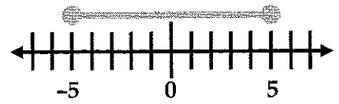

**b.** An adult male Nile crocodile may easily exceed 500 pounds and many adults reach 2,200 pounds.

$500 < p \le 2200$

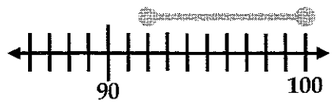



**c.** The weather report called for low temperatures ranging from –5°F to 5°F.

*The inequality is $-5 \le t \le 5$. The graph is:*




**12.** Write the symbolic inequality and a real-world example to match the graph of the inequality.




*The inequality is $92 \le a \le 100$. Answers for the real–world application will vary. A sample response: "To obtain an A in a class, you must have an average between at least 92% and 100% on all your work."*

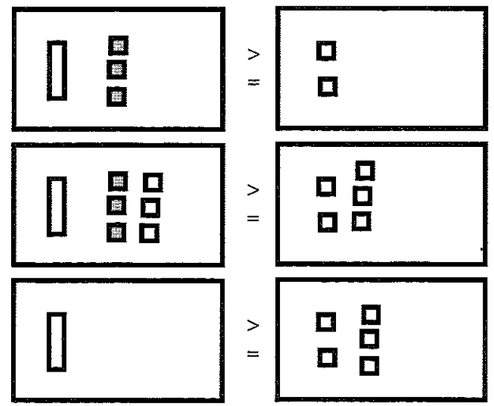


**Teacher Reference**

**Activity 2**

In this activity we will expand on solving one-variable equations to solving basic one-variable inequalities. The goal is for students to apply what they might know about writing and solving one-variable equations to writing and solving one-variable inequalities. Assign different groups one of the exercises between 1 and 4. Have the students write an inequality that matches the scenario and then solve the inequality to answer the question. Make sure the students are prepared to share their answers with the class. Have each group share how they wrote and solved an inequality for each scenario.

For the next part of this activity, students will use tiles and an inequality tile pad (located at end of unit) to solve inequalities using the Addition and Subtraction Properties of Inequalities. The class will continue to work in groups of four but as pairs on this activity. Write the inequality $x - 3 \ge 2$ on the inequality tile pad transparency and place the appropriate tiles on the pad to represent the inequality. Ask the class what they must do to isolate the variable on the inequality tile pad. Guide the class through adding three positive tiles to both sides of the pad to solve for the variable. As you model with the tiles also record the symbols. The zero pairs can be removed to yield the inequality $x \ge 5$.



$x - 3 \ge 2$

$x - 3 + 3 \ge 2 + 3$

$x \ge 5$

For Exercise 5, have a student volunteer model a second example, $y + 4 < -3$, on the overhead transparency using the overhead tiles while the class works on $y + 6 < 4$ on their dry–erase boards and the inequality tile pad. The class, and volunteer student, will need to cover up the equal sign for this example. The class, and volunteer student, will also need to orient the inequality tile pad to show < (less than). Lead a class discussion about adding and subtracting the same quantity from both sides of the equal sign and inequality sign. Ask the class, "How is adding or subtracting the same quantity from both sides of the inequality symbol similar to the Addition and Subtraction Properties of Equality?" Have the class agree on the definitions for the **Addition** and **Subtraction Property of Inequality** and write the definitions in their journal in the blank box provided. Have the class work in their groups, but as pairs, for Exercises 6 through 8. Have student volunteers share their results with the class. Sample definition for addition property of inequality: For all real numbers $a$, $b$, and c, if $a > b$, then $a + c > b + c$.

The second part of this activity will expand the properties of inequalities to the Multiplication and Division Properties of Inequalities. You will use the Inside–Outside Circles strategy for this part of the discovery activity. Split the class in half. Have half the class form an inner circle facing outward and the other half form an outside circle facing the inner circle of students. Have half of the inner circle students each choose a positive

number less than 10 and the other half a negative number greater than –10 and place the number on a dry-erase board. Make sure that the inner circle students are arranged as every other one has a positive number and every other one has a negative number. Have half of the outer circle of students write a greater than inequality with numbers only (such as $5 > 2$) and the other half write a less than inequality with numbers only (such as $-3 < 6$). Have the students on the inside pair up with a student on the outside. Tell the class to take the number from the inside student, of the student pair, and multiply both sides of the equality held by the outside student of the student pair. Determine if the inequality statement is true after the multiplication. Tell one of the student circles to rotate to the right by one student to form a new pair. Repeat the process until the students have had enough examples to determine that multiplying by a positive number is okay but that multiplying by a negative number results in a dilemma. You may also want the students to experiment with dividing.

Tell the class to go back to their groups of four to discuss and list their findings from the Inside–Outside Circles activity. Have the class answer Exercises 9 through 11 and have student volunteers share their results with the class. Ask the students guiding questions, such as "When did the inequality no longer become a true statement after multiplying or dividing both sides by some value?" Together as a class, write the definitions of the properties for multiplying and dividing both sides of an inequality. Have the class write these definitions in the blank box provided in their student journal.

Model solving the inequality $5x - 5 \geq 6x + 4$ by subtracting $6x$ from both sides so that eventually you will need to multiply both sides by a negative one which will cause the inequality to switch. Model solving $5x - 5 \geq 6x + 4$, while the class solves $8x + 8 < 9x - 7$ using the same process as yourself. Lead a discussion with the class on how solving inequalities is similar and different than solving equations. Ask guiding questions like:

- What are the similarities and differences in solving one-variable inequalities and one-variable equations?
- How did the order of operations assist you in solving one variable inequalities?

Have the students continue with Exercises 12 through 17 in their groups or pairs. Walk around the classroom and guide students as needed with the exercises or answer any questions they may have. Ask for student volunteers to share their results with the rest of the class.

Now you can introduce students to simple real-world applications for solving inequalities. Have the students work in groups to complete Exercises 18 through 21. Walk around the classroom and give suggestions as needed. If you feel that you need to model solving a real-world application you can use the following problem.

> The lead marching band for one of the Marti Gras parades needs a banner that is 5 feet in width. What are the possible lengths of the banner if the border has to be 50 feet or less?
>
> $P = 2l + 2w$ $\quad 2l + 2(5) \leq 50$ $\quad$ *Length would need to be less than or equal to 20 feet.*
>
> $l \leq 20$



**Activity 2** **SJ Page 62**

Your teacher will assign you a scenario below. You will write an inequality for the scenario then solve the inequality to answer the question.

1. The maximum safe load for each two-person chair on a ski lift is 425 pounds. The weight of one person on a chair is 208 pounds. What additional weight can the chair carry?

***Sample answer: The chair can hold up to an additional 217 pounds. Solving $x + 208 \leq 425$ for $x$ gives $x \leq 217$.***

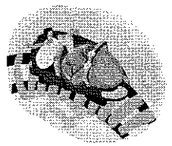
2. A cell phone plan allows a maximum of 750 minutes each month. If 147 minutes have been used this month, what range of minutes is left?

***Sample answers: There are up to an additional 603 minutes left. Solving $x + 147 \leq 750$ for $x$ gives $x \leq 603$.***

3. A family is planning a long trip across the United States that will take at least 36 hours to drive. They drove for 14 hours on the first day. How many hours do they have left to drive?

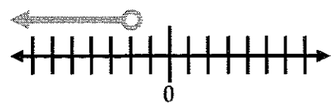
***Sample answer: There are at least 22 hours left to drive. Solving $x + 14 \geq 36$ for $x$ gives $x \geq 22$.***

4. Only customers 48 inches tall, or taller, can ride on most roller coasters. If Gabrielle is only 44.5 inches tall, how much does she have to grow to be able to ride the roller coaster?



***Sample answer: Gabrielle has to grow at least 3.5 inches. Solving $x + 44.5 \geq 48$ for $x$ gives $x \geq 3.5$.***

5. Create the inequality, $y + 6 < 4$, on the inequality tile pad using tiles. Solve the inequality with algebra tiles. Write a description of the solution and graph the solution.
***Look at students tiles as needed. The symbolic solution to the inequality is $y < -2$. The descriptive solution is "All real numbers less than a negative two." The graphed solution is:***




**Addition Property of Inequality –**

**Subtraction Property of Inequality –**

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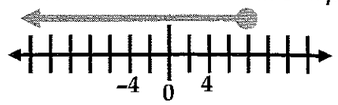
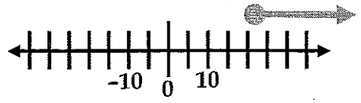
Use the **Addition Property of Inequality** and the **Subtraction Property of Inequality** to solve the inequalities in Exercises 6 through 8. Use tiles as needed.

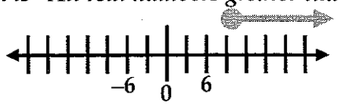
**6.** Solve and graph the solution to the inequality $4n \leq 3n + 8$. Write a descriptive solution to the inequality.
***The solution to the inequality is $n \leq 8$. The descriptive solution is "All real numbers less than or equal to a positive eight."***




**7.** Solve and graph the solution to the inequality $14 \leq n - 8$. Write a descriptive solution to the inequality.
***The solution to the inequality is $22 \leq n$. The descriptive solution is "All real numbers greater than or equal to a positive twenty–two."***




**8.** Solve and graph the solution to the inequality $13.5 + 7.2 + 9.8 + p > 40$. Write a descriptive solution to the inequality.
***The solution to the inequality is $p > 9.5$. The descriptive solution is "All real numbers greater than positive nine and one-half."***




The following questions are for the Inside–Outside Circles activity.

**9.** Describe what happened when you multiplied both sides of an inequality by a positive number.

***Answers will vary. A sample response might be: "When we multiplied both sides of an inequality by a positive number, the inequality was still a true statement."***

**10.** Describe what happened when you multiplied both sides of an inequality by a negative number.

***Answers will vary. A sample response might be: "When we multiplied both sides of an inequality by a negative number, the inequality was no longer a true statement."***

**11.** What might we do to make the inequality statement true when multiplying by a negative number? Do you think this would be true when dividing by a negative number?

***Answers will vary. A sample response might be: "Flip the inequality so that a greater than becomes a less than and a less than becomes a greater than." Yes. It is true for dividing by a negative number.***

| |
|---|
| **Multiplication Property of Inequality:** |
| **Division Property of Inequality:** |



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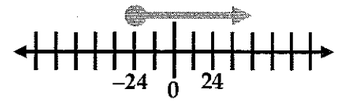
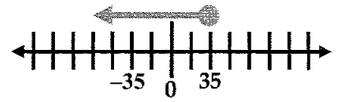
We will now solve inequalities using the **Multiplication Property of Inequality** and the **Division Property of Inequality**. Solve and graph the solutions to following inequalities. Be careful when multiplying or dividing both sides of an inequality by a negative number. Write a descriptive solution to the inequality.

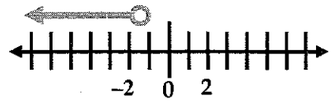
**12.** $-6g \leq 144$
***The solution to the inequality is $g \geq -24$. A descriptive solution is "All real numbers greater than or equal to negative twenty–four."***



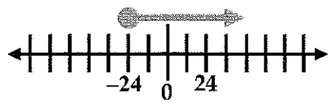

**13.** $\frac{y}{-5} \geq -7$
***The solution to the inequality is $y \leq 35$. A descriptive solution is "All real numbers less than or equal to thirty-five."***



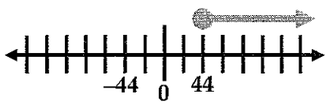

**14.** $21x < -28$
***The solution to the inequality is $x < \frac{-4}{3}$. A descriptive solution is "All real numbers less than or equal to negative four thirds." You did not divide by a negative number so the inequality symbol stays the same.***



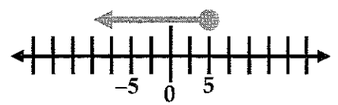

**15.** $\frac{5y}{8} \geq -15$
***The solution to the inequality is $y \geq -24$. A descriptive solution is "All real numbers greater than or equal to a negative twenty–four."***



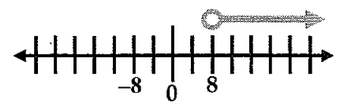

**16.** $\frac{-3q}{4} \leq -33$
***The solution to the inequality is $q \geq 44$. A descriptive solution is "All real numbers greater than or equal to forty–four."***



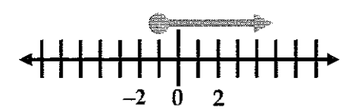

**15.** $\frac{2w}{5} + 7 \leq 9$
***The solution to the inequality is $w \leq 5$. A descriptive solution is "All real numbers less than or equal to five."***




**16.** $13k - 11 > 7k + 37$
***The solution to the inequality is $k > 8$. A descriptive solution is "All real numbers greater than eight."***




**17.** $7 + 3t \leq 2(t + 3) - 2(-1 - t)$
***The solution to the inequality is $-1 \leq t$. A descriptive solution is "All real numbers greater than or equal to a negative one."***



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**18.** The Booster Club needs to make a new school banner to display during championship tournaments. The length of the banner needs to be 15 feet. What are the possible widths, if the border of the banner can be no more than 42 feet? Write an inequality and then solve it.

$P = 2l + 2w$ $2(15) + 2w \le 42$ ***The width would need to be less than or equal to 6 feet.***

$w \le 6$

**19.** The cheerleaders would like to purchase small pompoms to pass out at the championship game. A case of 25 pompoms cost $20. What is the maximum number of cases that the cheerleaders can purchase if they can spend no more than $250?

***$20p \le 250$ $p \le 12.50$ They would need to purchase less than or equal to 12.50 cases. If it is assumed that partial cases cannot be purchased then they would need to purchase 12 or fewer cases.***

**20.** Girl Scout Cookies™ cost $3.50 per box. If Mercedes wants to serve them at her party but only has a budget of $20.00 for snacks, how many boxes can she purchase?

***$3.50p \le 20$ $p \le 5.71$ She would need to purchase less than 5.71 boxes, which works out to 5 or less boxes.***

**21.** Your MP3 player holds 8 GB of music. Each song you put on your MP3 averages 0.004 GB. Write an inequality that will let you know how many songs you can store on your MP3 player. How many songs at most can you store on your MP3 player?

***The inequality is $0.004s \le 8$. Solving the inequality yields $s \le$ 2,000 songs.***



**Teacher Reference**

**Activity 3**

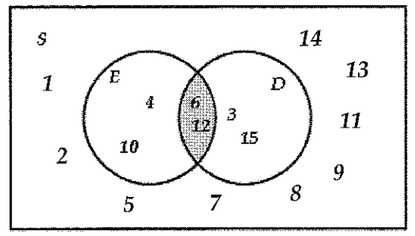
This activity will introduce the class to compound inequalities. The class has already worked with compound inequalities in the Setting the Stage perhaps without knowing they were compound inequalities. Lead a class discussion about inequalities of the form $4.50 \le x \le 7.50$. First talk about how this may represent a real-world concept such as books at the local book fair ranging in price from $4.50 to $7.50. Then ask guiding questions such as:

- Can we write the inequality $4.50 \le x \le 7.50$ into two separate inequalities with the same solution?
- Is the inequality $4.50 \le x \le 7.50$ the same as $4.50 \le x$ AND $x \le 7.50$? Why or why not?
- How might $4.50 \le x \le 7.50$ be the same as $4.50 \le x$ AND $x \le 7.50$?
- What might change in the inequality statement if the price of books all increased by a dollar?

Have student volunteers share their thoughts and reasoning on the questions. Tell the class that the word **AND** is a conjunction because it joins two inequalities into one and the solution must satisfy both inequalities simultaneously.

You can use the following Venn diagram concept to aid the class in a visual explanation of AND, which means intersection. The set $S$ contains the integers 1 through 15; set $E$ contains the integers {4, 6, 10, 12}; the set $D$ contains the integers {3, 6, 12, 15}. Have the students shade the area in the Venn diagram in Exercise 1 to represent the numbers that are only in $E$ **AND** $D$. The numbers 6 and 12 are in both sets. Ask the students questions to help them understand the connection of the term **and**, and the intersection of the two sets $E$ and $D$. Example questions could be:

- How does the Venn diagram represent the intersection of both sets?
- What part of the Venn diagram represents the term "and?"
- What would change in set E and D if there were no numbers in the intersection on the graph?

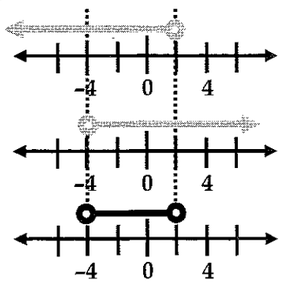



You may want to model creating a Venn diagram with the sets A {2, 4, 6, 8, 10, 12, 14} and B {3, 6, 9, 12} for the conjunction AND, while students complete Exercise 2. Visually inspect the student's results on their dry-erase boards. Create other example sets as needed.

Discuss the word "compound" with the class and ask them what it means. It is important for students to understand the term **compound** as it relates to inequalities. Now, model graphing the following compound inequality and have the class follow along on their dry-erase boards as you model.

$-4 < x$ and $x < 2$

It may be useful to show students how to use three number lines to graph the compound inequality. The third number line would represent the result of the compound inequality. You might want to use dashed lines projected onto the third number line to show the students the overlap of each inequality. Model how the original two inequalities can be written into a single inequality, $-4 < x < 2$, using the final results.



Model one of the sample exercises below while the class does a different sample exercise on their dry-erase boards. Have the class show you their results and inspect them visually.

**Sample Exercises**

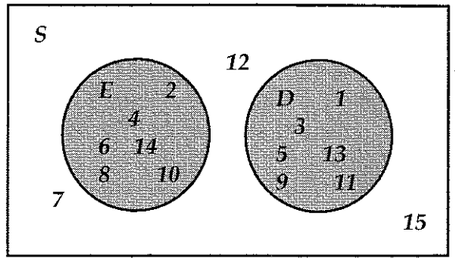
| | | |
|---|---|---|
| $-6 \leq x$ and $x < 12$ | $5 < x$ and $x \leq 9$ | $-5 \leq x < 11$ |
| $-3 < y < 5$ | $-2 \leq z + 3 < 8$ | $-3 < z + 2 < 7$ |

Have the students work in pairs for Exercises 3 through 8. For Exercise 6, explain or ask the class what the term **inclusive** means. Walk around the class facilitating and answering any questions or problems the students may come across. After the students have completed Exercises 3 through 8, ask for student volunteers to share their results and explain how they determined which terms were similar.

Now tell the class there is another type of compound inequality (also known as a conjunction, or a logical disjunction) related to the word **OR**. Lead a discussion about a real-world scenario such as discount prices at many restaurants are available to kids 10 and under and adults 55 years of age and older. A matching compound inequality would be $n \leq 10$ or $n \geq 55$. You can ask guiding questions such as:

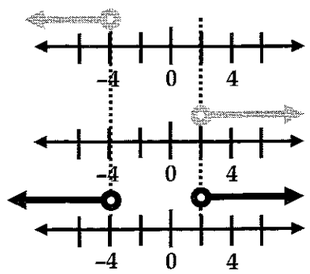
- Can we write the inequality $n \leq 10$ or $n \geq 55$ into one inequality statement?
- What might change in the inequality statement if the age of the adult changed to 60?
- What might the graph of the compound inequality look like?

You can model a Venn diagram example to aid the class in a visual explanation of OR. Have the students follow along with you using their dry–erase boards. The set $S$ contains the integers 1 through 15; set $E$ contains the integers {2, 4, 6, 8, 10, 14}; and the set $D$ contains the integers {1, 3, 5, 9, 11, 13}. Use a Venn diagram to display these sets. Shade which numbers are in $E$ or $D$?




Have the students hold up their dry–erase boards and visually check their results. The class should have all the numbers in both sets {1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 13, 14}. Ask the students if they understand the connection of the term **OR**, and the union of the two sets $E$ and $D$. The key here is for the students to see that we want everything in both sets. We will use this approach when graphing compound inequalities. For the following set of examples, model one set for OR with the students while the students work on another set for OR. Make sure to tell the students that we do not want duplicates. Visually inspect the student's results on their dry-erase boards. You may want to model creating a Venn diagram with the sets $A$ {2, 4, 6, 8, 10, 12, 14} and $B$ {3, 6, 9, 12} for the conjunction OR, while the students complete Exercise 9. Visually inspect the student's results on their dry-erase boards. Create other example sets as needed.

Now, model graphing $-4 > x$ or $x > 2$ while the class models $-6 > x$ or $x > 5$ for Exercise 10. The third number line would represent the combination of the both inequalities. You may want to use dashed lines to show the students that the graphed inequality is transposed to the solution graph.






Make sure the students understand that for a logical disjunction inequality, we include both graphs. We could also tell the students that **OR** represents a choice, hence the inclusion of both inequalities in the solution. If needed, model additional sample compound inequalities while the class does a different compound inequality. Have the class show you their results and inspect them visually.

**Sample exercises**

| | | | |
|---|---|---|---|
| $-6 \geq x$ or $x > 12$ | $5 > x$ or $x \geq 9$ | $-5 > y$ or $5 < y$ | $-3 > z + 2$ or $8 < x + 2$ |

Have the students work in pairs in their groups of four for Exercises 11 and 12. After the pairs have completed these exercises, have them check their answers with the other pair in the group. Ask for student volunteers to share their results and explanations. After the students have shared their results and explanations, have them complete Exercises 13 through 15 using the same process. Again, ask for student volunteers to share and explain their results.

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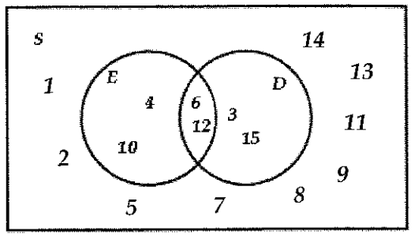
**Activity 3**

In this activity, you will investigate **compound inequalities**. The word **AND** is a conjunction because it joins two inequalities and the solution must satisfy both inequalities simultaneously.

To understand the concept of the conjunction **AND**, complete Exercises 1 and 2 as your teacher directs.

**1.** The Venn diagram on the right represents the 3 sets *S*, *E*, and *D*. The set *S* contains the integers 1 through 15; set *E* contains the integers {4, 6, 10, 12}; the set *D* contains the integers {3, 6, 12, 15}. Shade the area in the Venn diagram which represents the numbers that are both in sets *E* **AND** *D*.



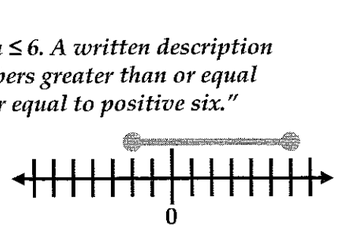

**2.** For the set *A* { –3, –1, 0, 1 , 3, 5} and *B* {–4, –3, –2, –1, 0, 1, 2, 3, 4}, draw a Venn diagram of the two sets and shade the area that contains the numbers that are common to both sets.

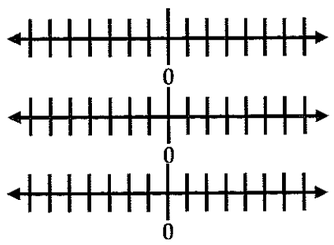
*See students' diagrams.*

For Exercises 3 and 4, graph the solution to the given compound inequality. Give a written description of the solution and write the inequalities as a compound inequality.

**3.** $a \leq 6$ AND $a \geq -2$

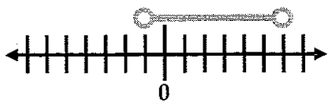
***The solution inequality is $-2 \leq a \leq 6$. A written description of the solution is "All real numbers greater than or equal to negative two and less than or equal to positive six."***

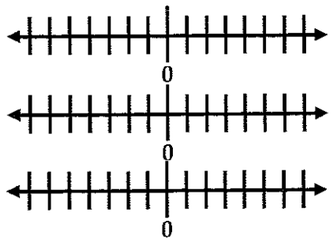



**4.** $-5 < x - 4$ AND $x - 4 < 2$

***The solution inequality is $-1 < x < 6$. A written description of the solution is "All real numbers greater than negative one and less than positive six."***



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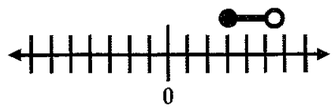
Now, we will concentrate on writing compound inequalities from various application problems. For Exercises 5 through 8, write a compound inequality for the given situation then graph the inequality.

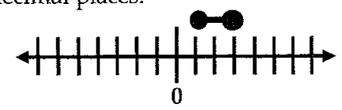
**5.** Starting salaries for college graduates range from $30,000, for educational services, to $56,000 for chemical engineering.

***The compound inequality is $\$30{,}000 \leq S < \$56{,}000$. NOTE: Each mark on the number line represents $10,000.***



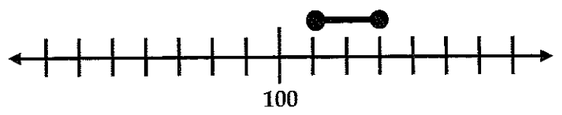

**6.** According to Hooke's Law, in order to displace a spring by a certain amount of centimeters from its "resting" length, a force *F*, in Newtons, needs to be applied to the spring. The equation for Hooke's Law is $F = kx$, where *k* represents the spring constant of elasticity. If the spring constant was calculated to be 1.68 and forces from 2 to 5 Newtons were applied to the spring, what will be the range of the displacements of the stretched spring? Round your answer to two decimal places.

***The compound inequality is $1.19 \leq x \leq 2.98$.***




**7.** The National Weather Service classifies hurricanes using the Saffir-Simpson Hurricane Scale. It is used to inform people of potential property damage and flooding caused by the storm's surge, winds, and rain. Wind speed is the major criteria that is used to categorize hurricanes. What is the range of wind speeds for a category 3 hurricane?

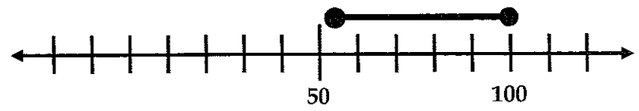
***Let w represent the wind speed, the inequality would be $111 \leq w \leq 130$.***




| Category Number | Wind Speed |
|---|---|
| 1 | 74–95 mph |
| 2 | 96–110 mph |
| 3 | 111–130 mph |
| 4 | 131–155mph |
| 5 | > 155 mph |

**8.** There are several stages of sleep which affect your heart rate. During the most restful period of sleep, your heart rate can be reduced by about 20%. During the most restless period of sleep, called REM, your heart rate can increase by about 50%. REM stands for "rapid eye movement," and it is during REM when your dreams take place. If your normal heart rate while you're awake is 66 beats per minute, what is the range of your heart rate while you are sleeping? Round your answer to the nearest heart beat.

***Let h represent the heart beats per minute. The inequality is $53 \leq h \leq 99$.***



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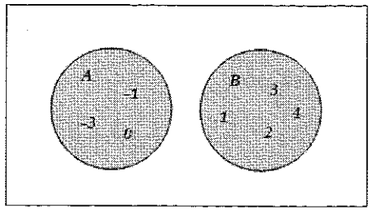
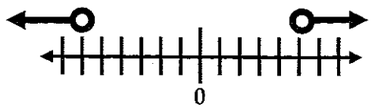
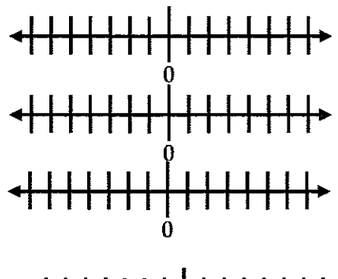
**9.** For sets $A$ {-3, -1, 0} and $B$ {1, 2, 3, 4}, draw a Venn diagram representing these sets and then shade in the portion representing $A$ OR $B$. Use curly brackets to list all the items in sets $A$ OR $B$.
***The items in set A OR set B are {–3, –1, 0, 1, 2, 3, 4}. The Venn diagram is***



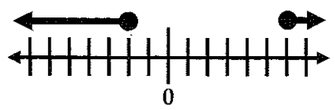

For Exercises 10 through 12, graph the solution to the given compound inequality. Give a written description of the solution.

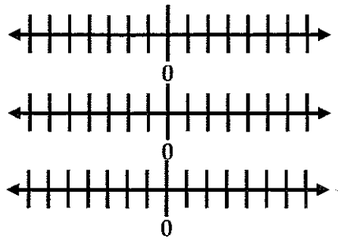
**10.** $-6 > x$ OR $x > 5$
***The description of the inequality is "All real numbers less than negative six or greater than five.***



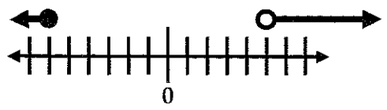



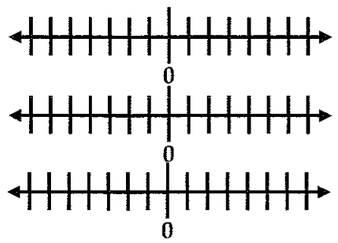

**11.** $a \geq 6$ OR $a \leq -2$
***The description of the inequality is "All real numbers less than or equal to negative two or all real numbers greater than or equal to six."***



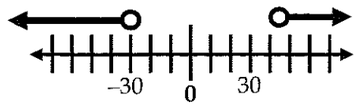




**12.** $-2 \geq x + 4$ OR $x + 4 > 9$
***The description of the solution is "All real numbers less than or equal to negative six or all real numbers greater than five."***




**13.** Write a compound inequality and description for the graph on the right, then write an application problem to go with your compound inequality.



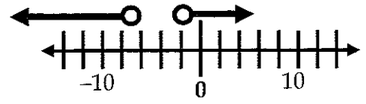

***The compound inequality is $x < -30$ OR $x > 45$. The description of the solution is "All real numbers less than negative thirty or all real numbers greater than forty–five." Application answers will vary.***

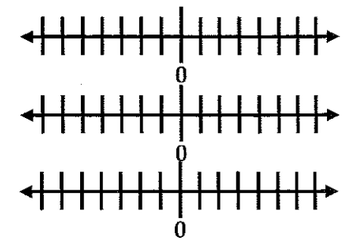


**SJ Page 69**

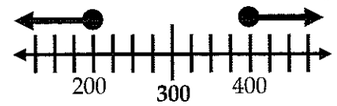
Now, let's write compound inequalities from various written and application type problems. For the following exercises, write a compound inequality for the given situation, find and graph its solution. Don't forget to label the variable if necessary.

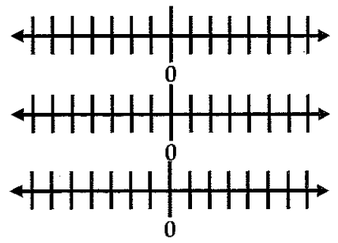
**14.** The product of –5 and a number is greater than 35 or less than 10.
***Let n be the number. The compound inequality is $-5n < 10$ OR $-5n > 35$ and the solution inequality is $n > -2$ OR $n < -7$.***




**15.** A store is offering a $30.00 mail-in rebate on all digital cameras costing at most $200.00 or at least $400.00.
***Let c be the cost of the camera. The solution inequality is $c < 200$ OR $c \geq 400$.***

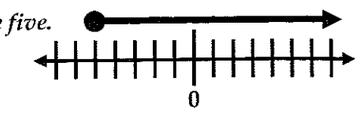


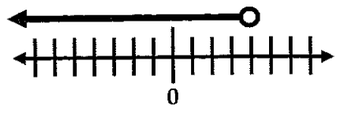
**SJ Page 70**
**Practice Exercises**

**1.** Graph the solutions to the following inequalities. Provide a written description of your solution.

**a.** $w \geq -5$ ***All real numbers greater than or equal to negative five.***

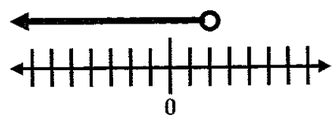



**b.** $x < 4$ ***All real numbers less than positive four.***



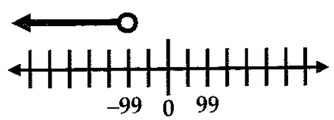

**c.** $80 - 24t > 32$

***The solution to the inequality is $t < 2$. The written description is "All real numbers less than two."***



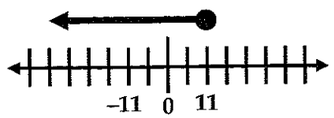

**d.** $\frac{-a}{11} > 9$

***The solution to the inequality is $a < -99$. The written description is "All real numbers less than negative ninety-nine."***



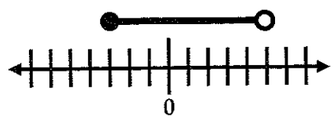

**e.** $4(y + 1) - 3(y - 5) \geq 3(y - 1)$

***The solution to the inequality is $y \leq 11$. The written description is "All real numbers less than or equal to eleven."***



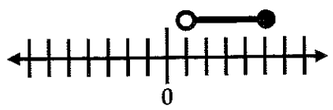

**f.** $x < 5$ AND $x \geq -3$

***The solution to the inequality is $-3 \leq x < 5$. The written description is "All real numbers less than five but greater than or equal to negative three."***




**g.** $-3 < d - 4 \leq 1$

***The solution inequality is $1 < d \leq 5$. The written description is "All real numbers less than or equal to five but greater than one."***



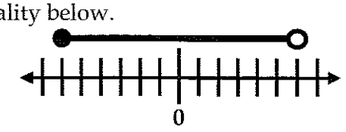

**2.** How are solutions to inequalities described in written and graphed formats?

***Answers will vary.***



**SJ Page 71**

**3.** Give a written and symbolic description of the graphed inequality below.




***"All real numbers greater than or equal to a negative six but less than a positive six."***
***$-6 \leq x < 6$***

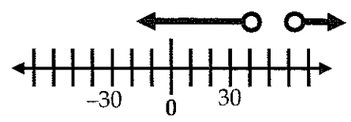
**4.** Different animals can hear different ranges of sound. Sound is measured in hertz which for hearing means vibrations per second. Sounds that an animal hears are actually caused by the air pressure changing back and forth very quickly. For example, on a piano the vibration that the note "*A*" above "middle C" makes is actually 440 hertz. This is caused by the string inside the piano vibrating 440 times per second which makes air pressure changes of 440 times per second that your ears sense. Humans can hear sounds as low as the 20 hertz and as high as 20,000 hertz. Bats can hear in the range of 20 to 200,000 hertz

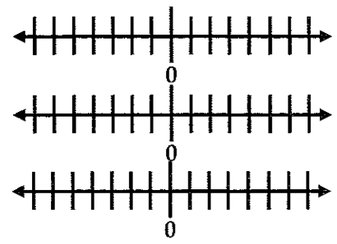
**a.** Write an equality that represents the range humans hear.
***$20 \leq h \leq 20{,}000$***

**b.** Write an equality that represents range that bats hear.
***$20 \leq b \leq 200{,}000$***

**c.** Use inequalities to represent the range of sounds bats can hear that a human cannot hear.
***Let $x$ represent the hertz range that bats can hear and humans cannot. The compound inequalities is $20{,}000 < x \leq 200{,}000$.***

**5.** Salmon sharks thrive in water temperatures which range from 41°F to 64°F. Write a compound inequality to represent the temperatures where the salmon shark may not thrive. Graph the compound inequality.
***Let $T$ represent the water temperature. The compound inequality is $T < 41$ OR $T > 64$.***



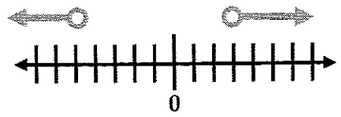



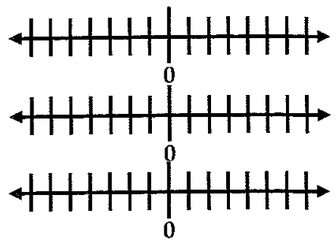
**SJ Page 72**

**6.** Solve the compound inequality $-3h + 4 > 19$ OR $7h - 3 > 18$ for $h$. Then graph the solution.

***h < –5 OR h > 3***

***The solution graph is:***

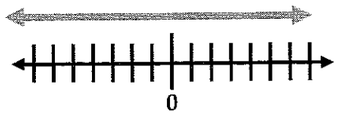


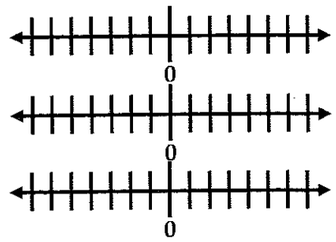




**7.** Solve the compound inequality $4t - 3 < 17$ OR $-8 \leq 3t - 2$

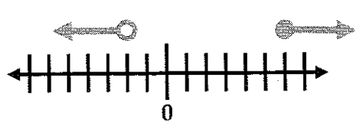
***t < 5 OR –2 ≤ t***

***The solution graph is:***




**8.** Write a compound inequality for the following graph. State the compound inequality in words.




***Sample response: The compound inequality is x < –2 OR x ≥ 6. All real numbers less than negative two or greater than or equal to positive six.***



**Outcome Sentences**

Graphing inequalities

Writing a description for an inequality

The difference between AND and OR for a compound inequality is

I know that an AND compound inequality

I know that an OR compound inequality

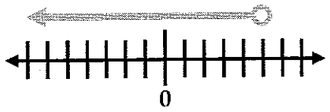
Determining an inequality from a graph

I am having trouble understanding

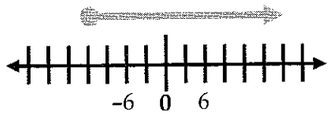
**Teacher Reference**

**Lesson 5 Quiz Answers**

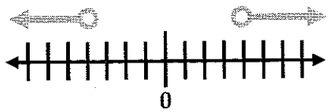
*1a.* *The solution is $x < 5$.*



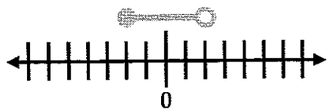

*1b.* *The solution is $x \geq -12$.*




*2a.* *The solution is $x > 4$ OR $x < -4$.*




*2b.* *The solution is $-2 \leq x < 2$.*




*3a.* *The inequality is $x > -5$.*

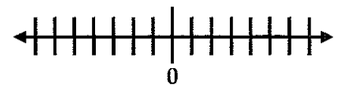
*3a.* *The inequality is $-9 < x \leq 21$.*



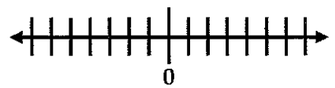
**Lesson 5 Quiz** **Name:** _____

**1.** Solve and graph the following inequalities.

**a.** $3x - 6 < 9$

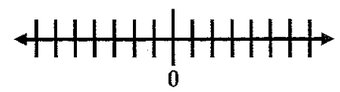



**b.** $\frac{-3x}{4} + 5 \leq 14$

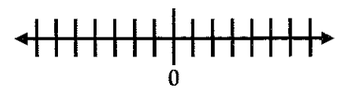



**2.** Solve and graph the following compound inequalities.

**a.** $-3x + 5 < -7$ OR $-2x - 3 > 5$

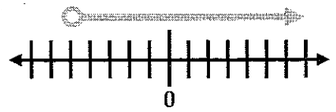



**b.** $6x - 4 < 8$ AND $4x - 1 \geq -9$

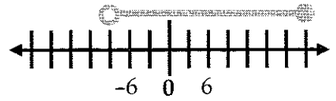



**3.** Write an inequality for the following graphs.

**a.**



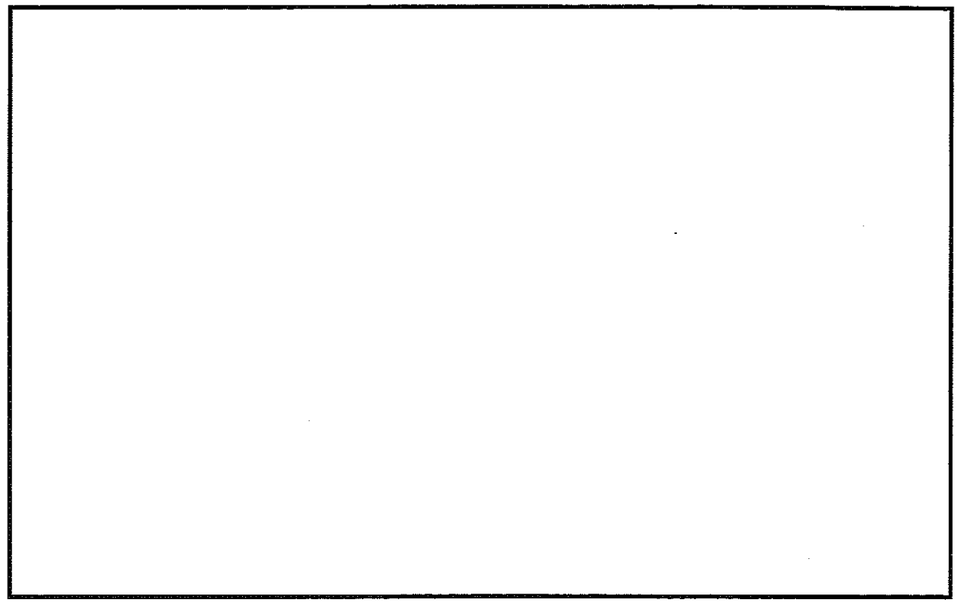

**b.**



**Number Line Transparency**



**Inequality Tile Pad Transparency**



II V