

Name: _____

This work may be exhibited if my name is removed: Yes No

Instructions. Always include explanations, so that other readers can tell what you did and why you did it. Never write outside the box. This problem is from the *Illustrative Mathematics Website*.

A-APR Trina's Triangles . Alice was having a conversation with her friend Trina, who had a discovery to share:

Pick any two integers. Look at the sum of their squares, the difference of their squares, and twice the product of the two integers you chose. Those three numbers are the sides of a right triangle.

Trina had tried this several times and found that it worked for every pair of integers she tried. However, she admitted that she wasn't sure whether this "trick" always works, or if there might be cases in which the trick doesn't work.

- (a) Investigate Trina's conjecture for several pairs of integers. Does her trick appear to work in all cases, or only in some cases?
- (b) If Trina's conjecture is true, then give a precise statement of the conjecture, using variables to represent the two chosen integers, and prove it. If the conjecture is not true, modify it so that it is a true statement, and prove the new statement.
- (c) Use Trina's trick to find an example of a right triangle in which all of the sides have integer length, all three sides are longer than 100 units, and the three side lengths do not have any common factors.