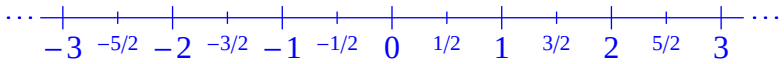
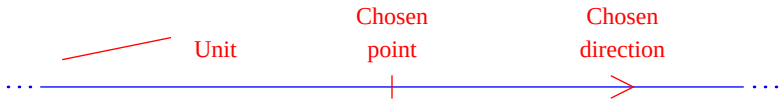
The Number Line M6302, June 5, 2012

The Number Line is the most important picture in mathematics.

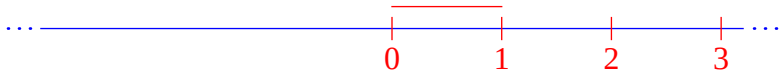


To make a number line, one decorates a naked line. Follow these steps:

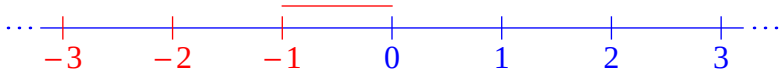
- STAGE I (setting things up)
  - Choose a unit of length.
  - Choose a point on the line.
  - Choose a direction from the point.



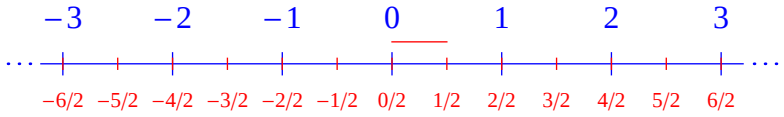
- STAGE II (labeling the integer points)
  - Lay copies of the unit end-to-end, starting from the chosen point and going in the chosen direction.
  - Label the endpoints of the units with the number of units one crosses to reach them.



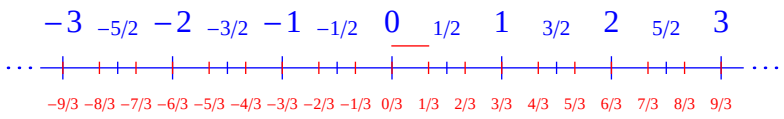
  - Lay copies of the unit, end-to-end, starting from the chosen point and going in the opposite of the chosen direction.
  - Label the endpoints of the units with the negative of the number of units one crosses to reach them.



- STAGE III (labeling rational points)
  - Divide the unit into 2 equal parts and use this to label the halves.



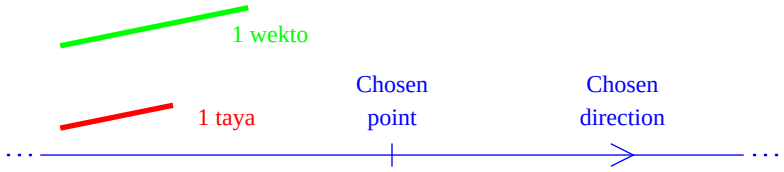
  - Divide the unit into 3 equal parts and use this to label the thirds.



  - Etc.

Two different units, with same origin and direction

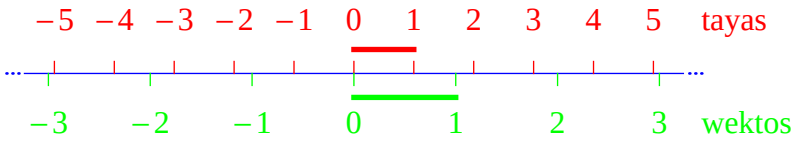
Suppose we label using two different units. We'll call one a *wekto* and the other a *taya*.



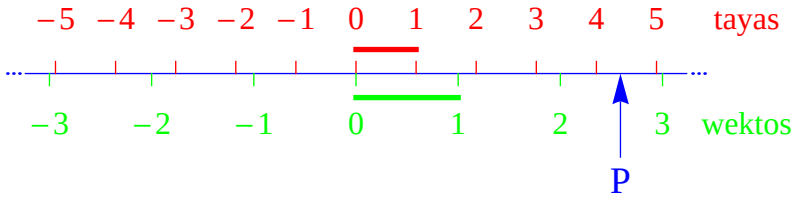
The relationship between wektos and tayas is:

$$10 \text{ wektos} = 17 \text{ tayas (or 1 wekto} = \tfrac{17}{10} \text{ tayas, or 1 taya} = \tfrac{10}{17} \text{ wektos).}$$

We can mark off wektos and tayas on the same line. We'll use the same origin and direction for both units.



Let P be a point on the line. Here, we picture P lying $\frac{22}{5} = 4.4$ tayas to the right of the origin.



The same P lies $\frac{44}{17} = 2.588\ldots$ wektos to the right of the origin. (Why?)

An arbitrary point X on this line has a coordinate in the taya system, and a coordinate in the wekto system. We let $t(X)$ denote the coordinate of X in the taya system. We let $w(X)$ denote the coordinate of X in the wekto system.

No matter where X lies on the line, it has a "taya coordinate" $t(X)$ and a "wekto coordinate" $w(X)$. Notice that $t(X)$ is a number—not a quantity with a label. So is $w(X)$. Both are numbers, and both numbers depend on the location of X. The point X lies $t(X)$ tayas to the right of the origin, and X lies $w(X)$ wektos to the right of the origin.

Problems

1) Suppose R is a point that has been chosen so that $t(R) = 51$. What is $w(R)$?
2) Suppose Q is a point that has been chosen so that $w(Q) = 36$. What is $t(Q)$?
3) How can I compute $t(X)$ from $w(X)$? $w(X)$ from $t(X)$?
4) Write an equation that relates $t(X)$ and $w(X)$. (Check it!)
5) What is the measure of a wekto in tayas? What is the measure of a taya in wektos?
6) How do the answers to 3), 4) and 5) relate to one another?