The Distributive Law June 7, 2012

If x, y and z are any numbers, then

$$x\,(y+z) = x\,y + x\,z.$$

Of course, it does not matter in what order we write the factors in a multiplication, so the distributive law implies that $(y+z)\,x = x\,y + x\,z$, and that $(y+z)\,x = y\,x + z\,x$, and so on. This proliferation of rules can become confusing. To minimize the burden on memory and attention, we only use the *left distributive law* (above) and the *right distributive law*:

$$(y+z)\,x = y\,x + z\,x.$$

Examples

a) $3\,(5+7) = 3(5) + 3(7)$

b) $365\,(x+8) = 365\,x + 365(8)$

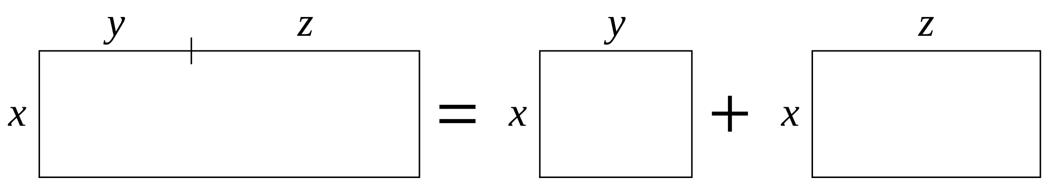
c) Keep the signs with the numbers: $(-3)\,(2+(-8)) = (-3)(2) + (-3)(-8)$

d) "Collecting like terms" is an instance of the distributive law, e.g., $5\,x + 7\,x = (5+7)\,x$.

e) The distributive law is what we use when we multiply binomials:

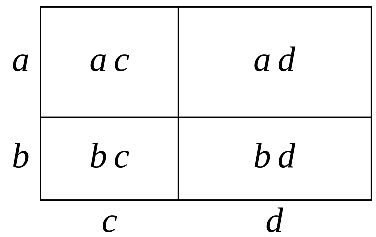
$$\begin{aligned}(x+5)\,(x+8) &= (x+5)\,x + (x+5)\,8 && \text{by the Left Distributive Law}\\ &= x\,x + 5\,x + x\,8 + 5\cdot 8 && \text{by the Right Distributive Law twice}\\ &= x^2 + 5\,x + 8\,x + 40 && \text{by the Commutative Law and arithmetic}\\ &= x^2 + 13\,x + 40 && \text{by the Righ Distributive Law and arithmetic}\end{aligned}$$

The distributive law can be understood as a statement about area. A rectangle that is x units high and $y+z$ units wide has the same area as two rectangles, one being x units high and y units wide and the other being x units high and z units wide.



If both the height and the width of a rectangle are cut into two pieces, the the whole rectangle is cut into four pieces; the area of the whole is the sum of the areas of the parts:

$$\begin{aligned}(a+b)\,(c+d) &= (a+b)\,c + (a+b)\,d \\ &= a\,c + b\,c + a\,d + b\,d\end{aligned}$$



Problems:

1) These problems examine how the distributive law is used when we expand products of polynomials.
   a) $(a+b+c)(d+e+f) =\ ?$
   b) Illustrate the application of the distributive law in part a) by using a rectangle, with one side consisting of three segments of lengths a, b and c, and the other side consisting of three segments of lengths d, e and f.
   c) How can you picture the terms of the expansion of $(a+b)^3$? How about $(a+b)^4$? $(a+b)^n$? (This is actually a prompt for an entire investigation.)
2) Where and how is the distributive law used in the multiplication algorithm that is taught in elementary school?
3) Some common errors involve over-generalization of the pattern observed in the distributive law. The following are not generally true:

$$(x+y)^2 = x^2 + y^2\ , \qquad \frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}\ , \qquad \sin(x+y) = \sin(x) + \sin(y).$$

   a) Have you observed similar errors in your students' work? Can you give examples? Have you ever questioned students who make these errors to find out what they believe about their work? What do they say?
   b) Are there any numbers x, y for which it *is* true that $(x+y)^2 = x^2 + y^2$? Are there any numbers x, y for which it *is* true that $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$?
4) What functions f do you know that satisfy the following law:

$$\text{for all numbers } x \text{ and } y,\ f(x+y) = f(x) + f(x)?$$

   If a function satisfies this law, what can you say about f? (This is a difficult question.)