

1:

- What division is Daniel referring to? You mention “conversion rate”? Define this term.
- Define “inches per centimeter” and “centimeters per inch”.
- What do you mean when you say “[He] used inches ...with quantitative value”?

2:

- You say that Daniel is making the magnitude statement: “.39 inch is about 1 cm.” Where does he say this?
- You say they can’t agree because Liana is making a number statement. What number statement is she making?

3:

- Is $(1 \text{ inch})/(2.54 \text{ cm})$ equal to 0.39? Or is it equal to 1?
- What is the “same left side” that you are referring to?
- Who is “representing a number”? Who is “representing a quantity”? What do you mean by “representing”?

4:

- I think you have made the difference between the two clear. Good.
- But I have a question. When you refer (lower left) to “the ratio of those two measurements” what are the things that are making the ratio? That is, if we have a ratio of one thing to another, then what is that one thing? And what is that other? Are they numbers? Magnitudes? Quantities? Or something else?

5:

- Is it true, as you say, that “an inch measured in terms of centimeters” equal to 0.39? How can this be?
- You say that Liana is “saying that it is just a conversion...”. What are you referring to when you say “it”?

6:

- I think you are on the right track; Liana is stating: “an inch measured by a centimeter is 2.54.”
- It is also true that .39 is a conversion factor. But so is 2.54.
- Is a conversion factor a number? Or a quantity (a number with a label)? If the latter, what is the label?
- Is $(1 \text{ inch})/(2.54 \text{ cm})$ equal to 0.39? Or is it equal to 1?

7:

- Is “1 inch = 2.54 cm” a “conversion”? What is your definition of “conversion”? (I would say that “1 inch = 2.54 cm” is just a fact, like “2 dimes = 4 nickels.”)
- What do you mean by saying, “if you take the number of each unit but not their magnitude”?

8:

- You are mentioning the right distinctions. But there are still some things that are obscure.
- When and why would you write, “in/cm = .39”? How is this related to “i/39 = c”. Are “in” and “i” referring to the same thing? “cm” and “c”?
- It’s true, isn’t it, that “inch/centimeter = 2.54”? What is this saying? Why isn’t it a division?
- Are both correct? “in/cm = .39” and “inch/centimeter = 2.54”. How can this be?

9:

- You refer to “quantities from his measurements”. Please explain what this means. Are you using the word “quantity” in the same way as the Common Core?
- What numbers is Liana referring to? Give an example. Let’s say Liana has measured some things in inches and in centimeters. Is she saying something about the numbers she got?

10:

- I think this is pretty clear. It is quite similar to what I would have written. The only challenge would be to make it clear to 9th-graders exactly what you mean by “variable.”

11:

- As you say, if we write “(9in/22.8cm) = 0.39 (in/cm)”, then we are using “0.39” the same way Daniel is. Your last sentence is very nice. Daniel is comparing two numbers that measure the same magnitude, where the two numbers arise from using two different units. Liana is comparing two magnitudes. I think this is pretty well put.

12:

- You are right about what Liana is saying.
- In the ration statement that you wrote, “inch/cm = 9/22.8”, what does “inch” refer to? A thing? A number? A quantity? A magnitude?

(13:)

- You are right about what Liana is saying.
- But what is Daniel saying?

14:

- What does it mean: “measure and deduce a conversion rate?”
- What does it mean: “form an equation which will be used as a ratio”?
- Is “1 inch for every 2.54 centimeters” a ratio?
- Would you say that a ratio is a relationship between two quantities, expressed as a number? Would you say, “The ratio of inch to centimeter is 2.54”?

15:

- Well OK, they are indeed are having a disagreement.
- You point out that 2.54 and .3947 are both conversion factors.
- Can you help Daniel and Liana understand one another?

16:

- You make a contrast between Liana talking about magnitudes (things that can be touched) and Daniel talking about numbers. This helps!
- You say that Daniel is concerned with “the exact numerical value”, and that he is referring to a “number of inches” and a “number of centimeters”.
- These are useful observations.

17:

- I find this pretty clear. Daniel, you say, is making statements that refer to the numbers that arise from measurements. When he uses the word “inches” he is referring to the number that is produced when a thing is measured by an inch. When Liana uses the word “inch”, she is referring to a small object, about as long as the second joint of my little finger.

18:

- Yes, one is making a number statement and one is making a magnitude statement. But which is which? How can you tell?

19:

- What do you mean by “conversion”? Do you mean “conversion factor”? Please define “conversion factor”.
- Are some conversions adjectives and some nouns?
- What can you say to make Daniel and Liana understand how they differ in the way they are using their words?