

ADJOINING AN IDENTITY ELEMENT TO A REDUCED ARCHIMEDEAN f -RING, II: ALGEBRAS

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This is joint work with Tony Hager

In "Part I" (presented at Ord05 (Oxford, MS)), we have discussed, for reduced archimedean f -rings, the canonical extension of such a ring, A , to one with identity, uA , and the class \mathbf{U} of u -extendable maps (i.e., homomorphisms which lift over the u 's to identity preserving homomorphisms). We showed that \mathbf{U} is a category and u becomes a functor from \mathbf{U} which is a monoreflection; the maps in \mathbf{U} were characterized. This paper addresses the interaction between our functor u , and v , the vector lattice monoreflection in archimedean ℓ -groups (due to Conrad and Bleier). In short, v restricts to a monoreflection of reduced archimedean f -rings into reduced archimedean f -algebras, $\psi \in \mathbf{U}$ if and only if $v\psi \in \mathbf{U}$, and vu is a monoreflection into reduced archimedean f -algebras with identity.

This work was motivated by the question put to us by G. Buskes at Ord05: what maps are o -extendable; i.e., extend over the orthomorphism rings? (The orthomorphism ring oA is a unital extension of uA , and any o -extendable map lies in \mathbf{U} .) While a complete answer seems quite complicated (if not hopelessly out of reach), here we shall identify a class of objects D for which $oD = vuD$ and all maps from D lie in \mathbf{U} , hence any map from D to a reduced archimedean f -algebra is o -extendable.