Seminar Notes: François Viète and his *Zetetica*

François Viète was born in 1540 in Poitou in west-central France. Viète was educated as a lawyer, and in the 1570s, having established a good reputation, he was invited by King Henry III to act as an adviser to the court in Paris. He served both Henry III and his successor Henry IV as privy councillor until his retirement in 1602. He died in 1603. The region of Viète's birth is also the region from which the Acadians emigrated to eastern Canada, beginning in 1604. The Cajuns of Louisiana are refugees of the British persecution of the Acadians in 1755–1763.

Viète studied and wrote about mathematics as a hobby. He is credited as the first mathematician to use letters as variables. One of his works, the *Five Books of Zetetica* (which was first published in 1591), contains numerous problems and solutions that illustrate his methods. Below, I have translated some of the problems into modern language.

Problems from *Zetetica: Book I*

1.1 Given the sum and the difference of two numbers, find the numbers.

Solution. Let the sum be denoted S and let the difference be denoted D. We are asked to find two numbers x and y such that $S = x + y$ and $D = x - y$. If we add the two equations together, we find $S + D = 2x$, so $x = (S + D)/2$. If we subtract, we find $S - D = 2y$, so $y = (S - D)/2$.

1.2 Given the difference between two numbers and their ratio, find the numbers.

1.3 Given the sum of two numbers and their ratio, find the numbers.

1.4 Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

1.5 Given two numbers that are greater than a third number and the ratio of their surpluses, find the third number.

1.6 Given two numbers, one greater than a third number and one less, and the ratio of the deficiency to the surplus, find the third number.

1.7 Divide a given length in such a way that a predefined part of the first segment plus a predefined part of the second will equal a prescribed number.

1.8 Divide a given length in such a way that a predefined part of the first segment subtracted from a predefined part of the second will equal a prescribed number.

1.9 Find two numbers with prescribed difference such that a predefined part of the first segment plus a predefined part of the second will equal a prescribed number.

1.10 Find two numbers with prescribed difference such that a predefined part of the first segment minus a predefined part of the second will equal a prescribed number.

Problems from *Zetetica: Book II*

2.1 Given the product and the ratio of two numbers, find the numbers.

2.2 Given the product of two numbers and the sum of their squares, find the numbers.

2.3 Given the product of two numbers and their difference, find the numbers.

2.4 Given the product of two numbers and their sum. find the numbers.

2.5 Given the difference between two numbers and the sum of their squares, find the numbers.

2.6 Given the sum of two numbers and the sum of their squares, find the numbers.

2.7 Given the difference between two numbers and the difference between their squares, find the numbers.

2.8 Given the sum of two numbers and the difference between their squares, find the numbers.

2.9 Given the product of two numbers and the difference between their squares, find the numbers.

2.10 Given *i*) the sum of the squares of two numbers plus their product (*i.e.*, $a^2 + b^2 + a\,b$) and *ii*) one of the numbers, find the other number.

2.11 Given *i*) the sum of the squares of two numbers plus their product and *ii*) the sum of the numbers, find the numbers.

2.12 Given *i*) the sum of the squares of two numbers plus their product (*i.e.*, $a^2 + b^2 + a\,b$) and *ii*) the product itself, find the other number.

2.13 Given the sum of the squares of two numbers and the difference of the squares, find the numbers.

2.14 Given the sum of the cubes of two numbers and the difference of the cubes, find the numbers.

2.15 Given the difference between the cubes of two numbers and the product of the numbers, find the numbers.

2.16 Given the sum of the cubes of two numbers and the product of the numbers, find the numbers.

2.17 Given the difference between the cubes of two numbers and the difference between the numbers, find the numbers. *(Four times the difference between the cubes minus the cube of the difference, if divided by three times the difference between the numbers, yields the square of the sum of the numbers.)*

2.18 Given the sum of the cubes of two numbers and the sum of the numbers, find the numbers. *(See 2.17.)*

2.19 Solve 17 another way. *(The difference between the cubes minus the cube of the difference, if divided by three times the difference between the numbers, yields the product of the numbers.)*

2.20 Solve 18, another way. *(See 2.19.)*

2.21 Find a and b in terms of $D = (a - b)(a^2 - b^2)$ and $S = (a + b)(a^2 + b^2)$.

2.22 Find a and b in terms of $S = a^2 + b^2$ and $R = \frac{a\,b}{(a-b)^2}$.

Prepared for the Louisiana Math and Science Teacher Institute at Louisiana State University. Supported by National Science Foundation Award Number 0928847.