

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let x and y represent known numbers and z represent an unknown number such that $y \leq x < z$. Let R denote the ratio of their deficiencies.

$$(z-y)R = \frac{z-x}{z-y}(z-y) \quad \bullet \text{ multiply both sides by } (z-y)$$

$$Rz - Ry = z - x \quad \bullet \text{ use the distributive law to distribute } R \text{ to } (z-y)$$

$\quad \quad -z \quad -z$

$$Rz - z - Ry = -x \quad \bullet \text{ Rearrange the equation so that all } z\text{'s are on one side of the equals sign.}$$

$\quad \quad +Ry \quad +Ry$

$$Rz - z = Ry - x$$

$$\frac{z(R-1)}{(R-1)} = \frac{Ry-x}{(R-1)} \quad \bullet \text{ Factor out } z$$

$$z = \frac{Ry-x}{(R-1)} \quad \bullet \text{ Divide both sides by } (R-1) \text{ to get } z \text{ by itself.}$$

Final solution in order to find the third number.

- Since the Ratio (R), x and y are all known to us, we can substitute these values into our equation and solve for z , the third number.

$$z = \frac{Ry - x}{R - 1}$$

Ratio
Ratio

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let x, y be the numbers and z be the third number
where $x \leq y < z$.

Let R be the ratio of deficiencies of x and y to z .

Then the deficiency of x and z is $z - x$, and
the deficiency of y and z is $z - y$.

Hence, the ratio of their deficiencies is $\frac{z-x}{z-y} = R$

$$z - x = R(z - y)$$

$$z - x = Rz - Ry$$

$$z - Rz = x - Ry$$

$$z(1 - R) = x - Ry$$

$$z = \frac{x - Ry}{1 - R}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let x and y be known values and z be an unknown value. Also let $x < z$ and $y < z$. Let r denote the ratio of their deficiencies.

$$\text{So } r = \frac{z-x}{z-y}$$

$$r(z-y) = z-x$$

$$rz - ry = z - x$$

$$rz - z = ry - x$$

$$z(r-1) = ry - x$$

$$z = \frac{ry - x}{r-1}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Problem: Given two numbers (x, y) that are less than a third number (z) and the ratio (R) of their deficiencies, find the third number.

Solution:

$$\begin{aligned} \text{Deficiency of } x &= z - x \\ \text{Deficiency of } y &= z - y \end{aligned} \quad R = \frac{z - x}{z - y}$$

Simplify in terms of z .

$$(z - y) \cdot R = \frac{z - x}{z - y} \cdot (z - y)$$

$$\begin{array}{r} Rz - Ry = z - x \\ + Ry \quad - Ry \end{array}$$

$$\begin{array}{r} Rz = Ry + z - x \\ - z \quad - z \end{array}$$

$$Rz - z = Ry - x$$

$$\frac{z(R - 1)}{(R - 1)} = \frac{Ry - x}{(R - 1)}$$

$$z = \frac{Ry - x}{R - 1}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

1st#	x
2nd#	y
3rd#	z

deficiency $\frac{z-x}{z-y}$

"Find z"

$$R = \frac{z-x}{z-y}$$

$$(z-y)R = z-x$$

$$Rz - Ry = z - x$$

$$Rz - z = Ry - x$$

$$z(R-1) = Ry - x$$

$$z = \frac{Ry - x}{R-1}$$

Identify your unknowns. Assume $z > x > y$.

The ratio is the deficiencies of z minus x divided by z minus y . Multiply $z-y$ on both sides of the equation. Distribute the R then factor out z . Finally, divide by $R-1$ on both side.

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

LET X BE THE SMALLEST NO.
 Y BE THE 2ND NO.
 Z BE THE 3RD + LARGEST NO.
 R BE THE RATIO OF DEFICIENCIES

SOLN:

• DEFICIENCY OF Z AND X :

$$\frac{Z-X}{}$$

• DEFICIENCY OF Z AND Y :

$$\frac{Z-Y}{}$$

• RATIO OF DEFICIENCIES : (SMALL/BIG)

$$R = \frac{Z-Y}{Z-X}$$

• SOLVING FOR Z :

$$Z-Y = R(Z-X)$$

$$Z-Y = RZ - RX$$

$$Z - RZ = Y - RX$$

$$Z(1-R) = Y - RX$$

$$Z = \frac{Y - RX}{1-R}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

There are three numbers a , b , and c where c is greater than either a or b . The differences between c and b and the difference between c and a are known as the ratio R .

$\frac{c-b}{c-a}$ as given in the problem.

Since $R = \frac{c-b}{c-a}$. Our third number is what we need to find, so we are solving this equation for c .

$$R = \frac{c-b}{c-a}$$

$$R(c-a) = c-b$$

$$Rc - Ra = c - b$$

$$Rc - c = Ra - b$$

$$c(R-1) = Ra - b$$

$$c = \frac{Ra - b}{R - 1}$$

this is our
third number

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let x , y , and z be the three numbers. Let x and y be the 2 known numbers and z be the unknown number. Assume that x is greater than y .

$$y < x < z$$

$$\text{Ratio: } R = \frac{z-x}{z-y}$$

Find z :

$$R = \frac{z-x}{z-y}$$

$$R(z-y) = z-x$$

$$Rz - Ry = z - x$$

$$Rz - z = Ry - x$$

$$\frac{z(R-1)}{(R-1)} = \frac{Ry-x}{(R-1)}$$

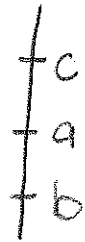
$$\frac{z(R-1)}{(R-1)} = \frac{Ry-x}{(R-1)}$$

$$z = \frac{Ry-x}{R-1}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

We begin with 3 number that we will call $a, b,$ and c . Let's say c is greater than a and a is greater than b as shown in the graph below.



Then their deficiencies would be $c-a$ and $c-b$. We do not know what c is but we do know the ratio of the deficiencies is. Therefore we can set up the following

$$R = \frac{c-a}{c-b}$$

We do not know c but now we can simply solve for c .

$$(c-b)R = (c-a)$$

$$cR - bR = c - a$$

$$cR - c = bR - a$$

$$c(R-1) = bR - a$$

$$c = \frac{bR - a}{R - 1}$$

We now know what c is.

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let the two known numbers be x and y , and a third number z that is unknown. $x < z$ and $y < z$.
The ratio of the deficiencies can be written as

$$\frac{z-x}{z-y} = R$$

$$z-x = R(z-y)$$

$$z-x = Rz - Ry$$

$$z - Rz - x = -Ry$$

$$z - Rz = -Ry + x$$

$$z(1-R) = -Ry + x$$

$$z = \frac{-Ry + x}{(1-R)}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let $x, y, \& z$ be unknown. We know that $x \leq y < z$

$$(z-y)R = \frac{z-x}{z-y} \quad (z=y) \quad \begin{array}{l} \text{Solve for } z \\ \text{Multiply both sides by} \\ (z-y) \end{array}$$

$$R(z-y) = z-x$$

$$Rz - Ry = z - x$$

$$Rz - z = Ry - x \quad \leftarrow \text{Get the } z\text{'s on one side}$$

$$\frac{z(R-1)}{R-1} = \frac{Ry-x}{R-1} \quad \begin{array}{l} \text{factor out the } z \\ \& \text{Divide by } (R-1) \end{array}$$

$$\text{to set } z = \frac{Ry-x}{R-1}$$

$$z = \frac{Ry-x}{R-1}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let x, y, z be three numbers such that
 $x < \frac{1}{3}z$ and $y < \frac{1}{3}z$. Also $\frac{z-x}{z-y} = R$.
Find the z (third number).

- Label

x = first number

z = (third number)

y = second number

R = ratio of their deficiencies (x and y and z)

So,

$$z - x = (z - y)R$$

$$z - x = zR - Ry$$

$$z - zR = -Ry - x$$

$$\frac{z(1-R)}{(1-R)} = \frac{-Ry - x}{(1-R)}$$

$$z = \frac{-Ry - x}{1-R}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let a and b be known numbers, and let x be the unknown number such that $a \leq b < x$. Let R represent the ratio of their deficiencies.

The deficiency of b is $x - b$, the deficiency of a is $x - a$; thus their ratio is $\frac{x-b}{x-a}$. So:

$$\text{(a)} \quad \frac{x-b}{x-a} = R(x-a)$$

$$\begin{array}{r} x-b = Rx - Ra \\ +b \qquad \qquad -Ra \end{array}$$

$$\begin{array}{r} x - Rx = -Ra + b \\ -Rx \quad -Rx \end{array}$$

$$x - Rx = -Ra + b$$

$$\frac{x(1-R)}{(1-R)} = \frac{-Ra + b}{(1-R)}$$

$$x = \frac{-Ra + b}{1-R}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

$$R = \frac{z-x}{z-y}$$

z = Largest #
 x = middle # (known) ratio
 y = smallest # (known)
 R = known

$$z-y \left(R = \frac{z-x}{z-y} \right) \quad \cancel{z-y} \quad \text{Multiply to eliminate denominator}$$

$$R(z-y) = z-x$$

Distribute R

$$Rz - Ry = z - x$$

Bring z to same side

$$Rz - Ry - z = -x + Ry$$

Bring y to other side

$$Rz - z = -x + Ry$$

Factor out z

$$\frac{z(R-1)}{R-1} = \frac{-x + Ry}{R-1}$$

Divide each side by $(R-1)$

$$z = \frac{-x + Ry}{R-1}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

Let x and y be known values and z be an unknown. Let R denote the ratio of their deficiencies.

So $x < z$ and $y < z$. Solve for z !

$$(z-y)(R) = \frac{(z-x)(z-y)}{(z-x)} \quad * \text{multiply both sides by } z-y$$

$$Rz - Ry = z - x \quad * \text{get } z \text{ by itself}$$

$$Rz - z = Ry - x \quad * \text{factor out } z$$

$$\frac{z(R-1)}{R-1} = \frac{Ry-x}{R-1} \quad * \text{divide both sides by } (R-1)$$

$$* z = \frac{Ry-x}{R-1}$$

$$z = \frac{Ry-x}{R-1}$$

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

known: $x, y, \text{ ratio}$
 unknown: z

Find third number, z

given:
 $x < z$
 $y < z$
 $R = \frac{z-x}{z-y}$

$$(z-y)R = \frac{z-x}{z-y} \cdot (z-y)$$

$$(z-y)R = z-x$$

$$Rz - Ry = z - x$$

$$-z + Ry \quad +Ry$$

$$Rz - z = Ry - x$$

$$\frac{z(R-1)}{(R-1)} = \frac{Ry-x}{(R-1)}$$

$$z = \frac{Ry-x}{R-1}$$

First, I looked at my ratio $R = \frac{z-x}{z-y}$. I then multiplied both sides by $(z-y)$. This gave me $(z-y)R = z-x$. I distributed the R , giving me

$Rz - Ry = z - x$. I need to get z by itself, so I added Ry to both sides. Then subtract z from each side. $Rz - z = Ry - x$

using the distributive property I "pulled out" the z . $z(R-1) = Ry - x$
 I divide each side by $(R-1)$ so $z = \frac{Ry-x}{R-1}$

I will test solutions to see if it works.

$x=1$
 $y=2$
 $\frac{R(2)-1}{R-1} = z$

- how can I plug in without knowing z . I can make up numbers for x, y but not R without knowing z .

Question: where does the $x < z, y < z$ come into play?

Show how to set up variables and equations to translate the following problem into algebra. Then solve the problem.

Given two numbers that are less than a third number and the ratio of their deficiencies, find the third number.

LET THE FIRST TWO NUMBERS BE REPRESENTED BY x & y AND THE THIRD BE z , THEN $z > x$ AND $z > y$. IF THEIR DEFICIENCIES IS HOW FAR x AND y ARE OFF FROM THEIR GREATER VALUE z THEN THEIR DEFICIENCIES CAN BE REPRESENTED AS $(z-x)$ AND $(z-y)$ THEREFORE THE RATIO OF THEIR DEFICIENCIES, CALLED D

$$D = \frac{z-x}{z-y} \text{ USING MULTIPLICATION PROP. OF EQUATION}$$

$$D(z-y) = z-x \text{ USING DISTRIBUTIVE PROP}$$

$$Dz - Dy = z-x \text{ BY ADD AND SUB PROP. OF EQ.}$$

$$Dz - z = Dy - x \text{ WE CAN THEN FACTOR FROM LEFT HAND SIDE OF EQUATION}$$

$$z(D-1) = Dy - x \text{ AND FINALLY WE CAN}$$

SHOW THE VALUE OF THE THIRD NUMBER, z WITH DIVISION TO BE!

$$z = \frac{Dy - x}{D - 1}$$