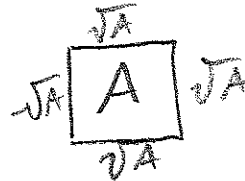


**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

LET "A" REPRESENT THE AREA OF A SQUARE AND "P" ITS PERIMETER

$$P = 4\sqrt{A}$$



THE AREA OF A SQUARE IS A SIDE SQUARED SO A SIDE MUST BE EQUAL TO THE SQUARE ROOT OF ITS AREA. FOUR SIDES MEANS WE MULTIPLY THIS ROOT BY 4.

2. Express the surface area of a cube as a function of its volume.

LET "SA" REPRESENT THE SURFACE AREA OF A CUBE AND "V" ITS VOLUME

$$SA = 6 \cdot \sqrt[3]{V^2} = 6V^{2/3}$$

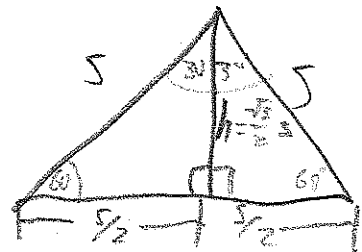


VOLUME OF A CUBE IS ONE OF ITS SIDES CUBED. SO THE CUBE ROOT OF THE VOLUME IS THE LENGTH OF EACH SIDE. SQUARING THIS LENGTH WILL GIVE THE AREA AND SINCE THERE ARE 6 FACES WE MULTIPLY THIS BY 6.

3. Express the area of an equilateral triangle as a function of the length of a side.

LET "A" REPRESENT THE AREA OF AN EQUILATERAL TRIANGLE AND "S" ITS SIDE LENGTHS.

$$A = \frac{1}{2}(S) \cdot \frac{\sqrt{3}}{2}(S) = \frac{\sqrt{3}}{4} S^2$$



THE AREA OF A TRIANGLE IS GIVEN BY  $\frac{1}{2} b \cdot h$  WHERE "b" IS THE BASE AND "h" ITS PERPENDICULAR HEIGHT WHICH WE KNOW FROM SPECIAL 30-60-90'S IS  $\frac{\sqrt{3}}{2} \times \text{HYPOTENUSE}$ .

1a

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.

WHAT DO WE KNOW ABOUT A SQUARE? IT HAS FOUR SIDES OF ALL THE SAME LENGTH, CALL THIS LENGTH "S"

<p>WE KNOW THE PERIMETER IS THE SUM OF ALL SIDES AROUND AN OBJECT SO TO FIND A SQUARE'S PERIMETER WE WOULD ADD "S" FOUR TIMES WHICH COULD SIMPLIFY AS "4S"</p> $P = 4s$	<p>WE KNOW AREA IS LENGTH TIMES WIDTH AND IN A SQUARE THE LENGTH AND WIDTH ARE BOTH "S" SO "S" TIMES "S" CAN BE SIMPLIFIED AS <math>s^2</math></p> $A = s^2$
---	--

HERE WE HAVE TWO FUNCTIONS IN TERMS OF "S". HOW CAN WE STATE "P" IN TERMS OF "A"? TO DO THAT LOOK AT AREA BACKWARDS. IF  $A = s^2 \Rightarrow s = \sqrt{A}$ .

SO IF  $P = 4s$  IT FOLLOWS THAT  $P = 4\sqrt{A}$  BECAUSE "S" IS EQUIVALENT TO " $\sqrt{A}$ ".

5. Express the surface area of a cube as a function of its volume.

WHAT DO WE KNOW ABOUT A CUBE? IT HAS LENGTH, WIDTH, AND DEPTH AND LIKE A SQUARE THEY ARE ALL THE SAME, CALL THIS "S" DIST.

<p>THE VOLUME CAN BE FOUND BY LENGTH X WIDTH X DEPTH</p> $V = s^3$	<p>EXAMINE THE SIX FACES OF A CUBE. THEY ARE ALL SQUARES OF LENGTH "S" AND WIDTH "S" SO EACH HAS AN AREA OF <math>s^2</math> AND ALL 6 GIVES A SURFACE AREA OF <math>6s^2</math>.</p> $SA = 6s^2$
--	---

HERE WE HAVE TWO FUNCTIONS IN TERMS OF "S". IF WE WANT TO EXPRESS SURFACE AREA, "SA", IN TERMS OF VOLUME, "V", WE MUST INVERT VOLUME'S FORMULA  $V = s^3 \Rightarrow s = \sqrt[3]{V}$  SINCE "S" IS THE SAME IN BOTH EQUATIONS WE CAN SUBSTITUTE "S" AS  $\sqrt[3]{V}$  TO GET  $SA = 6(\sqrt[3]{V})^2 = 6\sqrt[3]{V^2} = 6V^{2/3}$ .

6. Express the area of an equilateral triangle as a function of the length of a side.

AN EQUILATERAL TRIANGLE HAS THREE EQUAL SIDES. IT'S AREA CAN BE FOUND BY  $\frac{1}{2}bh$ . WE KNOW THE BASE IS THE SAME LENGTH AS THE SIDES, CALL IT "S". BUT HOW HIGH IS THE TRIANGLE? IT'S NOT "S" BUT THAT IS THE DIAGONAL LENGTH, EXAMINE THE RIGHT TRIANGLE MADE BY THE PERPENDICULAR BASE AND HEIGHT. WE KNOW THE BASE IS DIVIDED IN HALF " $\frac{s}{2}$ " AND THE DIAGONAL IS JUST "S". BY PYTHAGORAS' THEOREM

$$h^2 + \left(\frac{s}{2}\right)^2 = s^2 \Rightarrow h^2 = s^2 - \left(\frac{s}{2}\right)^2 \Rightarrow h^2 = \frac{4}{4}s^2 - \frac{1}{4}s^2 \Rightarrow h^2 = \frac{3}{4}s^2 \Rightarrow h = \sqrt{\frac{3}{4}s^2}$$

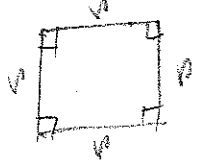
WHICH SIMPLIFIES TO  $h = \frac{\sqrt{3}}{2}s$  FINALLY WE CAN SHOW

$$A = \frac{1}{2} s \cdot \frac{\sqrt{3}}{2} s = \frac{\sqrt{3}}{4} s^2 \Rightarrow A = \frac{\sqrt{3}}{4} s^2$$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

ILLUSTRATION:



SOLN:

$$P = 4s$$

$$A = s^2$$

$$s = \sqrt{A}$$

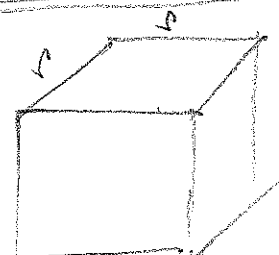
LET:  
 $s$  - SIDE of SQUARE  
 $A$  - AREA of SQUARE  
 $P$  - PERIMETER  
 $f(A)$  - PERIMETER of SQUARE as a func. of the AREA

$$P = 4\sqrt{A}$$

$$P = f(A) = 4\sqrt{A}$$

2. Express the surface area of a cube as a function of its volume.

ILLUSTRATION:



SOLN:

$$S.A. = 6s^2$$

$$V = s^3$$

$$s = \sqrt[3]{V}$$

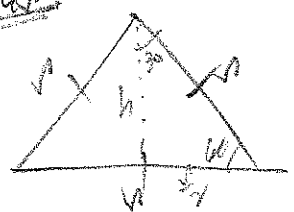
THEREFORE:

$$S.A. = 6\sqrt[3]{V^2} \rightarrow f(V) = 6\sqrt[3]{V^2}$$

LET:  $s$  - SIDE of CUBE  
 $S.A.$  - SURFACE AREA  
 $V$  - VOLUME  
 $f(V)$  - SURFACE AREA as a func. of Volume

3. Express the area of an equilateral triangle as a function of the length of a side.

ILLUSTRATION:



SOLN:

$$h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$$

$$= \sqrt{\frac{3s^2}{4}}$$

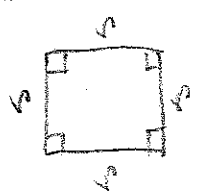
$$h = \frac{s}{\sqrt{3}} = \frac{s\sqrt{3}}{3}$$

LET:  
 $s$  - SIDE of TRIANGLE  
 $A$  - AREA " "  
 $f(s)$  - AREA " " as a function of the side

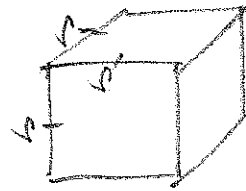
$$f(s) = \frac{s^2\sqrt{3}}{4}$$

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

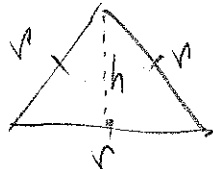
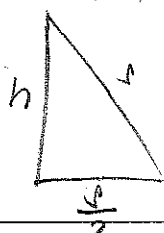
4. Express the perimeter of a square as a function of its area.

<p><u>PROBLEM ILLUSTRATION:</u></p>  <p><u>LET:</u></p> <p>s - SIDE of SQUARE  p - PERIMETER of SQUARE  A - AREA of SQUARE  f(A) - PERIMETER of SQUARE as a function of the AREA</p>	<p><u>SOL'N:</u></p> $P = 4s \quad \leftarrow \text{EQ. (1)}$ $A = s^2$ $s = \sqrt{A} \quad \leftarrow \text{EQ. (2)}$ <p><u>COMBINE EQ (1) + (2):</u></p> $P = 4\sqrt{A}$ <p>SINCE <math>P = f(A)</math>, THEREFORE</p> $f(A) = 4\sqrt{A}$
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5. Express the surface area of a cube as a function of its volume.

<p><u>PROBLEM ILLUSTRATION:</u></p>  <p><u>LET:</u></p> <p>s - SIDE of CUBE  S.A. - SURFACE AREA of CUBE  V - VOLUME of CUBE  f(V) - SURFACE AREA of a CUBE as a function of the VOLUME</p>	<p><u>SOL'N:</u></p> $S.A. = 6s^2 \quad \leftarrow \text{EQ (1)}$ $V = s^3$ $s = \sqrt[3]{V} \quad \leftarrow \text{EQ (2)}$ <p><u>COMBINE EQ (1) + (2)</u></p> $S.A. = 6(\sqrt[3]{V})^2 = 6\sqrt[3]{V^2}$ <p>SINCE <math>S.A. = f(V)</math>, THEREFORE</p> $f(V) = 6\sqrt[3]{V^2}$
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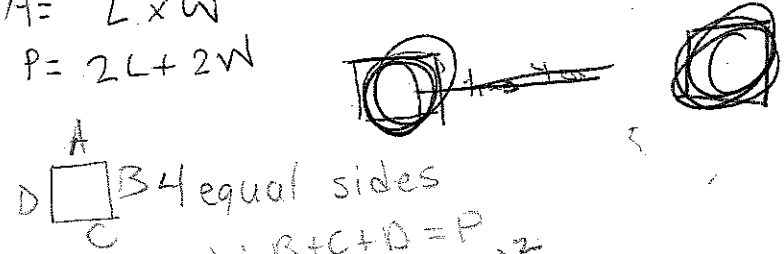
6. Express the area of an equilateral triangle as a function of the length of a side.

<p><u>PROBLEM ILLUSTRATION:</u></p>  <p><u>LET:</u></p> <p>s - SIDE of TRIANGLE  h - HEIGHT of TRIANGLE  A - AREA of TRIANGLE  f(s) - " " " " as a function of side  b - base of TRIANGLE</p>	<p><u>SOL'N:</u></p> $\text{EQ (1)} \rightarrow A = \frac{b \cdot h}{2}$ <p>BY INSPECTION, WE SEE 'b' = s.  HOWEVER, THERE IS A NEED TO SOLVE FOR 'h' IN TERMS OF 's':</p> <p>CONSIDER HALF of THE EQUI. TRIANGLE:</p>  <p>USE PYTHAGOREAN THEOREM TO SOLVE h</p> $h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$ $h = \sqrt{\frac{3s^2}{4}} = \frac{\sqrt{3}}{2}s$ <p>REPLACING 'b' AND 'h' IN TERMS OF 's'  RESULT TO: <math>A = \frac{s \cdot \frac{\sqrt{3}}{2}s}{2} = \frac{\sqrt{3}}{4}s^2</math></p> <p>EQ. (2)</p>
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**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

$A = L \times W$   
 $P = 2L + 2W$



$A$   
 $D$   $B$   $C$  4 equal sides  
 $A + B + C + D = P$   
 $A + B + C + D = A^2$

2. Express the surface area of a cube as a function of its volume.

Surface Area =  $6s^2$   
 $V = s^3$   
 $SA = 6\sqrt[3]{V^2} \rightarrow f(V) = 6\sqrt[3]{V^2}$   
 $f(V) = \text{surface area as a function of volume}$

3. Express the area of an equilateral triangle as a function of the length of a side.

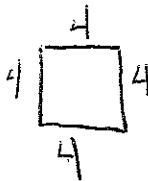
$A + B + C = E$   
 $A = B$   
 $B = C$   
 $A = C$

$A = s^2 \left( \frac{\sqrt{3}}{4} \right)$

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.

$P = 4s$  or  $2L + 2W$  or  $A + B + C + D$   
 $A^2 = \sqrt{A} = P$



$4 \times 4 = 4^2 = A = 16^2 = \sqrt{16^2}$   
 $4(2) + 4(2) = 16 = P$   
 $4(4) =$

There is a relationship of a function in a square b/t perimeter and area where the  $\sqrt{A} = P$

5. Express the surface area of a cube as a function of its volume.

$SA = 6s^2$      $SA = 6\sqrt[3]{V^2}$      $f(V) = 6\sqrt[3]{V^2}$   
 $V = s^3$

you can express the surface area of a cube as a function of its volume by relating it to the length of edge. SA of cube is  $6s^2$ .  
 $V = s^3$  Raise the volume equation to  $V^{2/3}$ .  
 Plug  $V^{2/3}$  into the SA formula  
 $A = 6s^{2/3}$

6. Express the area of an equilateral triangle as a function of the length of a side.

$A = s^2 \left( \frac{\sqrt{3}}{4} \right)$

~~$A = \frac{1}{2}bh$~~      $f(L) = \text{length of any side in equilateral triangle}$

$A = \frac{1}{2}bh$  In an equilateral triangle all sides have the same length, so the base or height may be used for any of the 3 sides, but will still produce the same answer.


When you divide an equilateral triangle in half you get 2 identical triangles, which are both right triangles. The sides of the equilateral triangle become the hypotenuses of the two right triangles. The base is  $\frac{1}{2}$  the length. To find the area we need to find the height. We find height by pythagorean theorem.

3b

When you divide an equilateral triangle in half you get 2 identical triangles, which are both right triangles. The sides of the equilateral triangle become the hypotenuses of the two right triangles. The base is  $\frac{1}{2}$  the length. To find the area we need to find the height. We find height by pythagorean theorem.

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.




$A = s^2$   
 $P = 4s$   
 $P(P) = 4\sqrt{A}$

• Connect the operations to the area formula  
 $A = s^2 \quad s = \sqrt{s^2} = \sqrt{A}$   
 $P = 4s = 4\sqrt{A}$

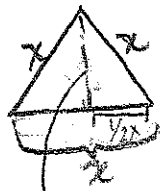
**Review**  
 • square 4 equal sides (s)  
 • Perimeter (P) =  $s + s + s + s = 4s$   
 • Area (A) =  $l \cdot w$  or  $s \cdot s = s^2$   
 • We are trying to right a formula for perimeter using the formula for area  
 • Identify common occurrences for P and A  $\rightarrow$  s and  $s^2$   
 • Identify the operation that converts  $s^2$  to s  $\sqrt{s^2} = s$

2. Express the surface area of a cube as a function of its volume.



$V = s^3$   
 $P = 6s^2$   
 $P(P) = 6s^2\sqrt[3]{V}$   
 or  $6(\sqrt[3]{V})^2$

3. Express the area of an equilateral triangle as a function of the length of a side.




$(\frac{1}{2}x)^2 + h^2 = x^2$   
 $\frac{1}{4}x^2 + h^2 = x^2$   
 $\frac{4}{4}x^2 - \frac{1}{4}x^2 = h^2$   
 $\sqrt{\frac{3}{4}x^2} = h^2$   
 $\frac{\sqrt{3}}{2}x = h$  (height of triangle)

$A = \frac{1}{2}bh \quad b=x \quad h = \frac{\sqrt{3}}{2}x$   
 $= \frac{1}{2}(x)(\frac{\sqrt{3}}{2}x)$   
 $= \frac{1 \cdot x \cdot \sqrt{3} \cdot x}{2 \cdot 1 \cdot 2} =$   
 $= \frac{(\sqrt{3})x^2}{4}$

$A(A) = \frac{x^2\sqrt{3}}{4}$

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader. (9<sup>th</sup> grader)

4. Express the perimeter of a square as a function of its area.



$s = \text{length of one side}$

$P = s + s + s + s = 4s$

$A = s^2$


ANSWER:

$f(P) = 4\sqrt{A}$

- Recall, square all sides equal, perimeter add up lengths of all sides, and area square one side.
- Let  $s = \text{length of one side}$ ,  $P = \text{perimeter}$ ,  $A = \text{Area}$
- Goal to write the perimeter formula in terms of the area
- $P = 4s$  Area  $= s^2$ ; identify what they have in common:  $s$  and  $s^2$
- Identify/perform an operation that makes them equal.  $s = \sqrt{s^2}$
- Replace the variable in  $P$  with the newly defined operation  $P = 4\sqrt{s^2}$

• Rewrite in terms of  $A$ :  $P = 4\sqrt{s^2} = 4\sqrt{A}$

5. Express the surface area of a cube as a function of its volume.



$s = \text{length of one side}$

$V = s^3$

$SA = 6s^2$


ANSWER

$6(\sqrt[3]{V})^2$

$6s^2\sqrt{V}$

- Recall: cube all edges equal, cube has 6 faces, Volume = edge cubed surface area = 6(edge)squared
- Let  $s = \text{length of one side}$ ,  $V = \text{volume}$ ,  $SA = \text{surface area}$
- Goal: to write the surface area formula in terms of the volume
- $V = s^3$ ,  $SA = 6s^2$ ; identify what they have in common:  $s^3$  and  $s^2$
- Identify/perform an operation(s) that makes them equal:  $\sqrt[3]{s^3} = s$  and  $(s)^2 = s^2$  so  $(\sqrt[3]{s^3})^2 = s^2$
- Replace the variable in  $SA$  with the newly defined operation:  $SA = 6(\sqrt[3]{s^3})^2$
- Rewrite formula in terms of  $V$ :  $6(\sqrt[3]{s^3})^2 = 6(\sqrt[3]{V})^2$

6. Express the area of an equilateral triangle as a function of the length of a side.



$A = \frac{1}{2}bh$

$b = x$

$h = \frac{\sqrt{3}}{2}x$

$A = \frac{\sqrt{3}}{4}x^2$

- Recall: Equilateral all sides equal, height is perpendicular bisector of the base, and height creates two congruent right triangles
- Let  $x = \text{side length}$ ,  $A = \text{area}$ ,  $b = \text{base}$ ,  $h = \text{height}$
- Goal to write the Area formula in terms of the side length
- $A = \frac{1}{2}bh$ ; identify the unknown: height
- Use pythagorean theorem to determine height (where base of right triangle =  $\frac{1}{2}x$  and hypotenuse =  $x$ )  
[work on back  $h = \frac{\sqrt{3}}{2}x$ ]
- Substitute variables into Area formula and solve

$$A = \frac{1}{2}bh = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right)$$

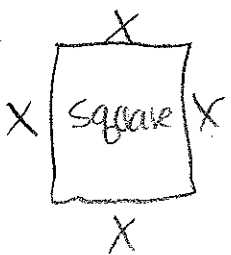
$$A = \frac{\sqrt{3}}{4}x^2 = \frac{1 \cdot x \cdot \sqrt{3} \cdot x}{2 \cdot 2} = \frac{\sqrt{3}x^2}{4} = \frac{\sqrt{3}}{4}x^2$$

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**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.



$$P = 4x$$

$$A = x^2$$

$$x = \sqrt{A}$$

$$P = 4\sqrt{A}$$

- Each side is equal lets call it  $x$ .

- perimeter is all sides added up.

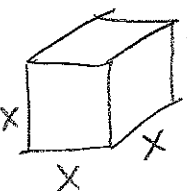
- Area is side squared

- if we add up all sides we get  $P = 4x$

- if we square sides  $A = x^2$  (solve for  $x$ )

$x = \sqrt{A}$  substitute  $P = 4(\sqrt{A})$

2. Express the surface area of a cube as a function of its volume.



$$SA = 6x^2$$

$$V = x^3$$

$$x = \sqrt[3]{V}$$

$$SA = 6(\sqrt[3]{V})^2$$

- Each side is equal. lets call it  $x$ .

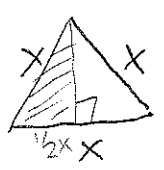
- surface area is area of all sides added up. So if the area of a square is  $x^2$  and we have six sides our surface area is  $6x^2$

- volume is  $lwh$ . so  $V = x \cdot x \cdot x = x^3$

- resolve volume for  $x$ , so  $x = \sqrt[3]{V}$

- substitute  $SA = 6(\sqrt[3]{V})^2$

3. Express the area of an equilateral triangle as a function of the length of a side.



$$A = \frac{1}{2}bh$$

$$b = x$$

$$h = \sqrt{\frac{1}{2}x^2}$$

$$A = \frac{1}{2}x\left(\sqrt{\frac{1}{2}x^2}\right)$$

$$A = \frac{1}{2}x\sqrt{\frac{1}{2}x^2}$$

$$A = \frac{1}{2\sqrt{2}}x^2$$

- equilateral triangle has equal sides lets call them  $x$ .

-  $A = \frac{1}{2}bh$  where  $b = x$ ,  $h = y$

- use the pythagorean theorem to find the height. (see work on left)

- talk through work on left as you do it

-  $A = \frac{1}{2\sqrt{2}}x^2$

$$x^2 = \frac{1}{2}x^2 + y^2$$

$$\frac{1}{2}x^2 = y^2$$

$$x^2 - \frac{1}{2}x^2 = y^2$$

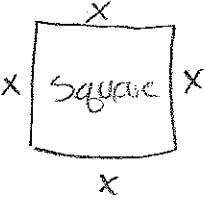
$$\frac{1}{2}x^2 = y^2$$

$$\sqrt{\frac{1}{2}x^2} = \sqrt{y^2}$$

$$y = \sqrt{\frac{1}{2}x^2}$$

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.



$P = x + x + x + x$   
 $P = 4x$   
 $A = l \cdot w$   
 $A = x \cdot x$   
 $A = x^2$   
 $\sqrt{A} = \sqrt{x^2}$ ,  $\sqrt{A} = x$   
 $P = 4\sqrt{A}$

- A square has 4 equal sides, so lets call each side  $x$ .

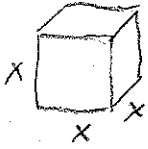
- The perimeter is all sides added up, so we add  $x + x + x + x$ . our  $P = 4x$

- Area is length  $\cdot$  width so  $x \cdot x$ ,  $A = x^2$

- we need to solve perimeter in terms of area, so if we solve for  $x$  in Area we take the square root of both sides  $\sqrt{A} = x$

- now substitute  $P = 4x$  where  $x = \sqrt{A}$   $P = 4\sqrt{A}$ !

5. Express the surface area of a cube as a function of its volume.



$A_s = x^2$   
 $SA = x^2 + x^2 + x^2 + x^2 + x^2 + x^2$   
 $SA = 6x^2$   
 $V = l \cdot w \cdot h$   
 $V = x \cdot x \cdot x$   
 $V = x^3$   
 $\sqrt[3]{V} = \sqrt[3]{x^3} = \sqrt[3]{V} = x$

- cube has all equal sides, lets call them  $x$ .

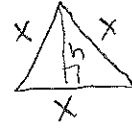
- Area of each side we already said is  $x^2$  in problem 1. If we want surface area we have to add up all six sides area

$x^2 + x^2 + x^2 + x^2 + x^2 + x^2 = SA = 6x^2$

- volume equals  $l \cdot w \cdot h$  which is  $x \cdot x \cdot x$  or  $V = x^3$

- if we want to solve surface area in terms of volume we have to solve volume for  $x$ . Take the cube root of each side,  $x = \sqrt[3]{V}$ , substitute  $SA = 6x^2$

6. Express the area of an equilateral triangle as a function of the length of a side.



$A = \frac{1}{2}bh$   
 $b = x$   
 $h = \frac{\sqrt{3}}{2}x$   
 $A = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}\right)x$   
 $A = \frac{\sqrt{3}}{4}x^2$

- an equilateral triangle has all equal sides lets call them  $x$ .

- the formula for Area of a triangle is  $A = \frac{1}{2}bh$  our  $b = x$ , but we dont know height.

- lets use pythagorean theorem to find height

(go through solving for height with work to the left) once we know height and base we can plug into our formula and we get  $A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x$  which will =  $A = \frac{\sqrt{3}}{4}x^2$

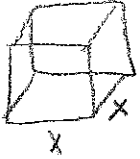
$x^2 = \left(\frac{1}{2}x\right)^2 + h^2$   
 $x^2 = \frac{1}{4}x^2 + h^2$   
 $\frac{1}{4}x^2 = \frac{1}{4}x^2$   
 $\sqrt{\frac{3}{4}x^2} = h$ ,  $\frac{\sqrt{3}}{2}x = h$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.


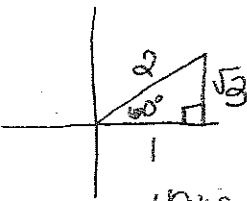
1. Express the perimeter of a square as a function of its area.

Let  $x =$  side of the square so area  $= x^2$ . By solving for  $x$  we obtain  $x = \sqrt{\text{area}} = \sqrt{A}$ . We know perimeter = the sum of the side lengths so  $P = 4x$ . By substitution we get  $P = 4\sqrt{\text{area}} = 4\sqrt{A}$ .

2. Express the surface area of a cube as a function of its volume.

  $SA = 2x^2 + 4x^2 = 6x^2$  We need to solve our surface area equation for  $x$  so that we can substitute it into the volume equation. We get  $x = \sqrt{\frac{SA}{6}}$ .  
 $V = x^3$   
 Therefore  $V = x^3 = \left(\sqrt{\frac{SA}{6}}\right)^3$

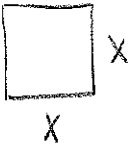
3. Express the area of an equilateral triangle as a function of the length of a side.

 Area of an equilateral triangle is equal to  $\frac{1}{2}x^2 \sin 60^\circ$ . We should also evaluate the  $\sin 60^\circ$  using right triangle trig which is equal to  $\frac{\sqrt{3}}{2}$ . Now we have  $A = \frac{1}{2}x^2 \left(\frac{\sqrt{3}}{2}\right)$ .  
 By simplifying we get  $A = \frac{\sqrt{3}}{4}x^2$ . We now must use techniques of solving equations to obtain  $x = \sqrt{\frac{4\sqrt{3}}{3}A}$ .

6a


**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.



We will let the length of the sides of our square equal  $x$ . The perimeter<sup>(P)</sup> is equal to the sum of the lengths of the sides of the figure so  $P = x + x + x + x = 4x$ . Area<sup>(A)</sup> is equal to the length of the base times the length of the height. Both of these are equal to  $x$  so  $A = x(x)$  or  $A = x^2$ . Now let's solve for  $x$  in our area equation by taking the square root of both sides so  $x = \sqrt{A}$ . By substitution we have  $P = 4x = 4\sqrt{A}$ .

5. Express the surface area of a cube as a function of its volume.



We let  $x =$  the length of the sides of our cube. We know surface area is equal to the sum of the areas of the faces & 2 bases so  $SA = 4x^2 + 2x^2 = 6x^2$ . Volume =  $l \times w \times h$  so  $V = x^3$ . To solve for  $x$  we take the cube root of both sides so  $x = \sqrt[3]{V}$ . Now we can substitute  $\sqrt[3]{V}$  for  $x$  into the surface area equation so,  $SA = 6(\sqrt[3]{V})^2$ .

6. Express the area of an equilateral triangle as a function of the length of a side.

If we recall our formula for area of a  $\Delta$  that says  $A = \frac{1}{2} (\text{side length}) (\text{side length}) (\sin(\text{included angle}))$  and know the properties of an equilateral  $\Delta$  this is an easy problem! All sides =  $x$  units & we know all angles are  $60^\circ$ . so  $A = \frac{1}{2} x(x) \sin 60^\circ$  or  $\frac{1}{2} x^2 \sin 60^\circ$ . We also with special right  $\Delta$ 's know  $\sin 60^\circ = \frac{\sqrt{3}}{2}$  so we can simplify to  $A = \frac{1}{2} x^2 \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} x^2 \text{ units}^2$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

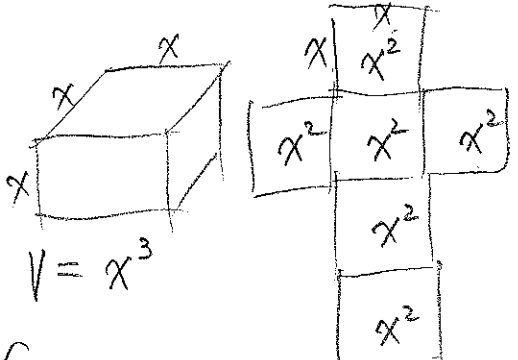
1. Express the perimeter of a square as a function of its area.  $P(A(x)) = ?$

Let  $x$  be the side of a square. Since a square has 4 equal sides, then the perimeter,  $P$ , is  $4x$ . Furthermore, the area of a square,  $A$ , is  $x^2$ . Because we are asked to express the perimeter of a square as a function of its area, we need to solve for  $x$  with  $A = x^2 \Rightarrow \sqrt{x^2 = A} \Rightarrow x = \pm\sqrt{A}$ .  $x = \sqrt{A}$  (b/c Area is positive)

$$\begin{aligned} P(x) &= 4x \\ A(x) &= x^2 \end{aligned}$$

$P = 4x$ , by substitution  $\Rightarrow P = 4\sqrt{A}$

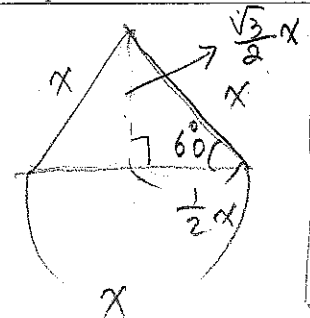
2. Express the surface area of a cube as a function of its volume.  $S(V(x)) = ?$



$V = x^3$  and  $SA = 6x^2$   
 $\sqrt[3]{V} = \sqrt[3]{x^3} \Rightarrow x = \sqrt[3]{V}$   
 By substitution,  $SA = 6(\sqrt[3]{V})^2$

(Let  $x$  be the side of a cube)

3. Express the area of an equilateral triangle as a function of the length of a side.



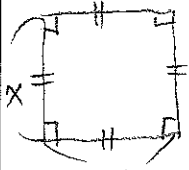
By using  $30^\circ-60^\circ-90^\circ$  special right  $\Delta$  ratio, we can find the height of the  $\Delta$  as  $\frac{\sqrt{3}}{2}x$ . Then the area of an equilateral  $\Delta$  is  $= (\frac{1}{2})(x)(\frac{\sqrt{3}}{2}x) = \frac{\sqrt{3}}{4}x^2$ .

$\therefore A \text{ of } \Delta_{\text{equal}} = \frac{\sqrt{3}}{4}x^2$

(Let  $x$  be the side of an equilateral  $\Delta$ .)

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.



Let  $x$  be the side of a square. We need to come up with two functions; perimeter function & area function, in order for us to answer this question.

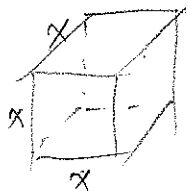
Since, a square has all 4 sides same length, then  $P = x + x + x + x = 4x$ , and  $A = (x)(x) = x^2$ .

Furthermore, we need to express the function of perimeter in terms of its area, we first need to solve for  $x$  from the area.

$A = x^2 \Rightarrow x = \pm \sqrt{A}$ , but  $x = \sqrt{A}$ , (b/c Area is always positive).

By substitution,  $P = (4)(\sqrt{A})$

5. Express the surface area of a cube as a function of its volume.

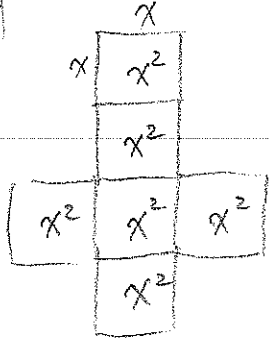


$V = (x)(x)(x) = x^3$ ;  $SA = x^2 + x^2 + x^2 + x^2 + x^2 + x^2 = 6x^2$

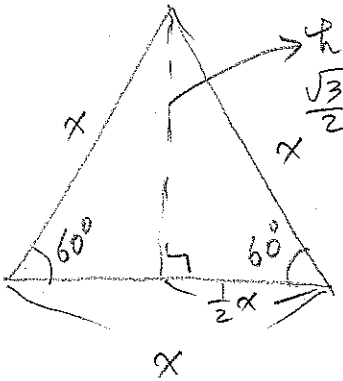
$\sqrt[3]{V} = \sqrt[3]{x^3} \Rightarrow x = \sqrt[3]{V}$

By substitution,  $SA = (6)(\sqrt[3]{V})^2$

(Let  $x$  be the side of a cube)



6. Express the area of an equilateral triangle as a function of the length of a side.



Let  $x$  be the length of a side of an equilateral  $\Delta$ .

Area of  $\Delta = \frac{1}{2}(\text{base})(\text{height})$ .

We know the base, but we do not know the height of this  $\Delta$ . The height of the  $\Delta$  can be found using Pythagorean Theorem.

$(\frac{1}{2}x)^2 + (h)^2 = (x)^2 \rightarrow h = \frac{\sqrt{3}}{2}x$

Area of  $\Delta = (\frac{1}{2})(x)(\frac{\sqrt{3}}{2}x) = \frac{\sqrt{3}}{4}x^2$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

$P = 4s$  The perimeter of a square is equal to the sides squared.  
 $P = s + s + s + s$  The perimeter of a square is also equal to the sum of its four sides.  
 \* Since all sides of a square are of equal length,  $l$  and  $w$  can be one variable  $s$   
 $A = lw$   
 $A = s^2 = P$

2. Express the surface area of a cube as a function of its volume.

$V = lwh$        $SA = 2lw + 2lh + 2wh =$   
 $V = 2(lwh)$        $SA = 2(lw + lh + wh)$

3. Express the area of an equilateral triangle as a function of the length of a side.

$A = \frac{1}{2}bh$        $P = s + s + s$   
 $A = \frac{1}{2}(P - 2s)(P - 2s)P - 2s = s$   
 $A = \frac{1}{2} - s(P - 2s)$   
 $A = \frac{1}{2} - sp + 2s^2$

8a

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.

Let's first understand what the statement is saying. Expressing the perimeter of a square as a function of its area means that we are going to use both the formula for area and perimeter of a square to derive a new formula, that combines the two.

List the formulas:

$P = 4s$   
 $A = s^2$

Since we want a formula that finds the perimeter, we will solve the area formula for  $s$  and substitute the  $s$  value into the formula for Perimeter.

$A = s^2$   
 $\sqrt{A} = \sqrt{s^2}$   
 $\sqrt{A} = s \rightarrow$  substitute into perimeter

$P = 4s$   
 $P = 4(\sqrt{A})$   
 $P = 4\sqrt{A}$

x when to use -  
 If given the area of a square, we can use  $P = 4\sqrt{A}$  to find the perimeter of a square.

5. Express the surface area of a cube as a function of its volume.

The above statement simply means that we will use both the formula for surface area and volume of a cube to derive a formula for the surface area that uses the volume of a cube.

List the formulas:

$SA = 6s^2$   
 $V = s^3$

Since we want a formula that finds the surface area of a cube, we will solve the volume formula for  $s$  and substitute the formula for  $s$  into the formula for surface area.

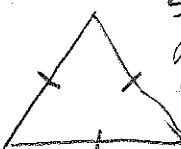
$V = s^3$   
 $\sqrt[3]{V} = \sqrt[3]{s^3}$   
 $\sqrt[3]{V} = s \rightarrow$  substitute into surface area

$SA = 6s^2$   
 $SA = 6(\sqrt[3]{V})^2$   
 $SA = 6(\sqrt[3]{V})^2$

x when to use -  
 If given the volume of a cube, we can use  $SA = 6(\sqrt[3]{V})^2$  to find the surface area of a cube.

6. Express the area of an equilateral triangle as a function of the length of a side.

Since we know that an equilateral triangle has side of equal length and the area of a triangle can be found using  $A = \frac{1}{2}bh$ , then we only need to determine what the height of an equilateral triangle is equal to. We will let each side be denoted by  $s$ . Using Pythagorean theorem we can find the height of the triangle.



$A = \frac{1}{2}sh$   
 Known Known

$a^2 + b^2 = c^2$   
 $a^2 + (\frac{1}{2}s)^2 = s^2$   
 $a^2 + \frac{1}{4}s^2 = s^2$   
 $a^2 = s^2 - \frac{1}{4}s^2$   
 $a^2 = \frac{3}{4}s^2$

$\sqrt{a^2} = \sqrt{\frac{3}{4}s^2}$   
 $a = \frac{1}{2}s\sqrt{3}$   
 $a$  is equal to the height of the triangle

Substitute into the original area formula:  
 $A = \frac{1}{2}s(\frac{1}{2}s\sqrt{3})$   
 $A = \frac{1}{4}s^2\sqrt{3}$



**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

The area ( $A$ ) of a square is defined to be  $S^2 = A$  where  $S$  is the measure of the side. The measure of the side is found to be  $S = \sqrt{A}$ . The perimeter of a square is the sum of the four congruent sides, therefore, since  $S = \sqrt{A}$ , then the perimeter can be expressed as a function of the area such that  $P = 4(\sqrt{A})$ .

2. Express the surface area of a cube as a function of its volume.

Surface area of a cube is defined to be  $SA = 6S^2$  where  $S$  is the measure of a side. The volume of a cube is defined to be  $V = S^3$ . The measure of the side of a cube can be expressed as an expression of the volume such that  $\sqrt[3]{V} = S$ . Hence, to express the surface area as a function of the volume,  $SA = 6(\sqrt[3]{V})^2$ .

3. Express the area of an equilateral triangle as a function of the length of a side.

The area of triangle is defined to be  $A = \frac{1}{2}bh$ . The base (side) of the triangle we will express as  $x$ , and the height will be expressed as a function of  $x$  under rules for special right triangles (30-60-90) to be  $x\sqrt{3}$ . Hence the area of an equilateral  $\Delta$  is found to be  $A = \frac{1}{2}x^2\sqrt{3}$

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.

The area of a square is found by multiplying the length <sup>and</sup> the width ~~and~~. Since this is a square the length will equal the width. We'll call that value  $x$ . So the area is  $x \cdot x$  or  $A = x^2$ . If we solve for  $A$  we get  $x = \sqrt{A}$ . The perimeter of the square is found by taking the sum of all four sides so  $P = x + x + x + x$  or  $P = 4x$ . But we know that  $x$  is equal to  $\sqrt{A}$  so we substitute in the place of  $x$  to get  $P = 4\sqrt{A}$

5. Express the surface area of a cube as a function of its volume.

The surface area of a cube is found to be the sum of the area for all six sides. We already know length = width so we will let  $x$  be our side measure. So our formula gives us  $SA = 6x^2$ . The volume of a cube is the length  $\cdot$  width  $\cdot$  height, but in a cube length = width = height, so we will use  $x$  again. Our formula for volume will be  $V = x^3$ . Solving for  $V$  we see that  $\sqrt[3]{V} = x$ . We will substitute this value for  $x$  back into our SA equation.  

$$SA = 6(\sqrt[3]{x})^2$$

6. Express the area of an equilateral triangle as a function of the length of a side.

An equilateral triangle has side measure of  $x$ . The area of a triangle is defined to be  $A = \frac{1}{2}(\text{base} \times \text{height})$ . We need to find the height. The height is always perpendicular to the base, which will create a right  $\Delta$ , and you know how to deal w/ right  $\Delta$ 's! Pythagorean Thm!  
 Our hypotenuse will be  $x$ , our short leg will be  $\frac{1}{2}x$  and our height will be  $h$ . So  $x^2 - \frac{1}{4}x^2 = h^2$ . Solving for  $h$  we get  $h = \frac{x\sqrt{3}}{2}$ . Plugging back into our equation  $A = \frac{1}{2}(x)(\frac{x\sqrt{3}}{2})$  so  
 Our answer is  $A = \frac{x^2\sqrt{3}}{4}$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

Let  $A = \text{Area}$

$$A = s^2, P = 4s$$

$$s = \sqrt{A}$$

$$P = 4\sqrt{A}$$

Area is side times side, while perimeter is the sum of the sides. Solve the Area formula for side, now we can use that new representation of a side in the perimeter formula to get the perimeter with Area as the variable; which is the same as saying perimeter as a function of Area.

2. Express the surface area of a cube as a function of its volume.

Let  $SA = \text{Surface Area}$ ,  $V = \text{Volume}$

$$SA = 6s^2$$

$$V = s^3$$

$$s = \sqrt[3]{V}$$

$$SA = 6(\sqrt[3]{V})^2$$

We know the Volume and Surface Area formulas. To get SA as a function of  $V$ , solve  $V$  for  $s$ , then plug in a new representation of  $s$  into SA formula.

3. Express the area of an equilateral triangle as a function of the length of a side.

Let  $s = \text{length of side}$ ,  $A = \text{Area}$

$$A = \frac{1}{2}bh$$

$$b = s$$

$$h^2 + \left(\frac{1}{2}s\right)^2 = s^2$$

$$h^2 = \frac{3}{4}s^2$$

$$h = \frac{\sqrt{3}}{2}s$$

$$A = \frac{\sqrt{3}}{4}s^2$$



Equilateral triangle has sides of same length. we can use  $A = \frac{1}{2}bh$  and the pythagorean theorem in order to get Area as a function of the sides. since the triangle is equilateral, the perpendicular height will bisect the side, then we just use pythagorean theorem to find  $h$  and  $b$ 's length of side, plug into formula.

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area. Let:  $A$  = Area,  $P$  = Perimeter,  $s$  = side.

$$A = s^2, P = 4s$$

$$\sqrt{A} = \sqrt{s^2}$$

$$\sqrt{A} = s$$

$$P = 4s$$

$$P = 4(\sqrt{A})$$

$$P = 4\sqrt{A}$$

First write your formulas. The question wants Perimeter as a function of Area, which means Area needs to be the variable. Solve the Area Formula for  $s$ . Now we can replace the  $s$  in the perimeter function with  $\sqrt{A}$ . This gives us the perimeter with  $A$  as the variable.

5. Express the surface area of a cube as a function of its volume. Let:  $SA$  = Surface Area,  $V$  = Volume,  $s$  = side.

$$SA = 6s^2, V = s^3$$

$$\sqrt[3]{V} = \sqrt[3]{s^3}$$

$$\sqrt[3]{V} = s$$

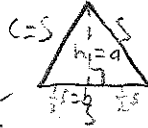
$$SA = 6(\sqrt[3]{V})^2 = 6\sqrt[3]{V^2}$$

$$SA = 6\sqrt[3]{V^2}$$

First write your formulas. We want Surface Area as a function of Volume, so solve the Volume formula for  $s$ . Now we can replace the  $s$  in the surface area formula with the expression  $\sqrt[3]{V}$  from the volume formula. Now we have Surface Area with  $V$  as the variable.

6. Express the area of an equilateral triangle as a function of the length of a side. Let:  $A$  = Area,  $b$  = base,  $h$  = height,  $s$  = side.

Pythagorean Theorem

$$A = \frac{1}{2}bh$$


$$a^2 + b^2 = c^2$$

$$h^2 + (\frac{1}{2}s)^2 = s^2$$

$$h^2 = s^2 - (\frac{1}{2}s)^2 = \frac{4}{4}s^2 - \frac{1}{4}s^2 = \frac{3}{4}s^2$$

$$h = \sqrt{\frac{3s^2}{4}} = \frac{\sqrt{3} \cdot \sqrt{s^2}}{\sqrt{4}} = \frac{\sqrt{3} \cdot s}{2}$$

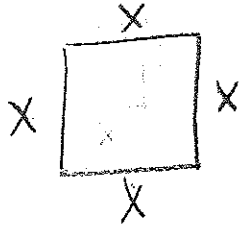
$$A = \frac{1}{2} \cdot s \cdot \frac{\sqrt{3} \cdot s}{2}$$

$$A = \frac{\sqrt{3} s^2}{4}$$

First write the formula. Now draw a picture, remember in equilateral triangles all sides are the same length. Draw the height of the triangle by drawing a perpendicular line, this cuts the triangle in half, so on each side of  $h$  we have  $\frac{1}{2}s$ . Look at the left half and use the Pythagorean theorem to solve for  $h$ . Realize that  $b$  is just a side and plug into the Area formula. Now Area is written as a function of the sides.

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 miutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

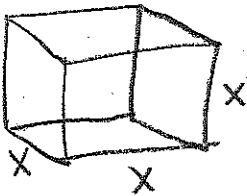


$$\text{area} = x \cdot x = x^2$$

$$\text{perimeter} = x + x + x + x = 4x$$

$$\text{perimeter} = 4 \sqrt{\text{area}}$$

2. Express the surface area of a cube as a function of its volume.

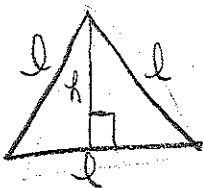


$$\text{surface area} = 6x^2$$

$$\text{volume} = x^3$$

$$6 \left( \sqrt[3]{\text{volume}} \right)^2 = \text{surface area}$$

3. Express the area of an equilateral triangle as a function of the length of a side.



$$l^2 = \left(\frac{1}{2}l\right)^2 + h^2 \quad (\text{height})$$

$$h^2 = l^2 - \frac{1}{4}l^2$$

$$a = \frac{1}{2}(bh)$$

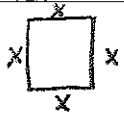
$$a = \frac{1}{2}(l) \left( l^2 - \frac{1}{4}l^2 \right)$$

$$a = \frac{1}{2} \left( l^3 - \frac{1}{4}l^3 \right) \Rightarrow \frac{1}{2}l^3 - \frac{1}{8}l^3$$

1/a

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

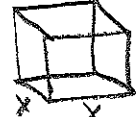
4. Express the perimeter of a square as a function of its area.



$A = x^2$   
 $P = 4x$   
 $\sqrt{x^2} = x$   
 $4x$   
 $4\sqrt{A}$

We know square has equal side, let's call each side  $x$ . Area is equal to  $x \cdot x = x^2$ . Our perimeter would be  $x + x + x + x = 4x$ . If we want to write our perimeter in terms of our area, then what would do to our  $x^2$  to make equal to our  $4x$ ? Well to get from  $x^2$  to  $x$  we need to take square root  $\sqrt{x^2} = x$ . Now we have  $x$  we can multiply to get  $4x$  so our function would be  $4\sqrt{\text{area}}$ .


5. Express the surface area of a cube as a function of its volume.



$A = 6x^2$   
 $V = x^3$   
 $x^3 \rightarrow x = \sqrt[3]{x^3}$   
 $x \rightarrow x^2 = (\sqrt[3]{x^3})^2$   
 $6 \cdot x^2$   
 $6(\sqrt[3]{x^3})^2$

A cube has 6 square sides. We know area of a square is  $x^2$  so then if we have 6 squares our surface area is  $6x^2$ . We get volume of a cube by multiplying length, width and height so  $x \cdot x \cdot x$  or  $x^3$ . We need to write  $6x^2$  in terms of  $x^3$ . So first we  $\sqrt[3]{x^3}$  to get  $x$ , now we have  $x$  so we need it squared so  $(\sqrt[3]{x^3})^2$  so we have  $x^2$  so we need to multiply by 6 to get  $6x^2$  so our function is  $6(\sqrt[3]{x^3})^2$ .

6. Express the area of an equilateral triangle as a function of the length of a side.



$l^2 = (\frac{1}{2}l)^2 + h^2$   
 $h^2 = l^2 - \frac{1}{4}l^2$   
 $h^2 = \frac{3}{4}l^2$   
 $h = \frac{\sqrt{3}}{2}l$

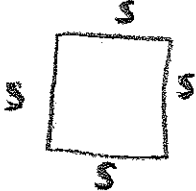
Equilateral all sides are equal, so all  $l$  to find area we need height ( $h$ ). Using Pyth. then  $l^2 = (\frac{1}{2}l)^2 + h^2$ . We can find  $h$  from this. Now we have height we can plug into area formula of  $\frac{1}{2}bh$ .

$\frac{1}{2}l(\frac{\sqrt{3}}{2}l) = \frac{\sqrt{3}l^2}{4}$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.


1. Express the perimeter of a square as a function of its area.

$P = 4s$      $\sqrt{A} = \sqrt{s^2}$   
 $P = 4\sqrt{A}$   
 $P(A) = 4\sqrt{A}$




2. Express the surface area of a cube as a function of its volume.

$SA = 6s^2$   
 $V = A_b s$   
 $V = s^2 s$   
 $V = \sqrt[3]{s^3}$



3. Express the area of an equilateral triangle as a function of the length of a side.

$A = \frac{1}{2}bh$



$A = \frac{1}{2}L(h)$

$A = \frac{1}{2}L \left( \frac{L\sqrt{3}}{2} \right) = \frac{A(L) \frac{L^2\sqrt{3}}{4}}{2}$

$h = \frac{L\sqrt{3}}{2}$

$A = \frac{1}{2}bh$   
 $\frac{1}{2}L(3\sqrt{3})$   
 $9\sqrt{3}$

$L^2 + \left(\frac{L}{2}\right)^2 = L^2 - \frac{1}{4}L^2$   
 $h = \frac{3L\sqrt{3}}{2}$   
 $A = \frac{1}{2}bh$   
 $A = \frac{1}{2}L \left( \frac{3L\sqrt{3}}{2} \right) = \frac{3L^2\sqrt{3}}{4}$

12a

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

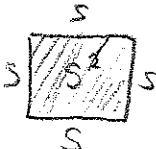
4. Express the perimeter of a square as a function of its area.

$$P = 4s$$

$$\sqrt{A} = \sqrt{s^2}$$

$$\sqrt{A} = s$$

$$P(A) = 4\sqrt{A}$$



Perimeter - the distance around the figure.

Area - the coverage of the figure. (space inside)

Say we're given the area of the sq. How do we find its perimeter. Find a way to deduct the space inside to just one side. To find area we take one length and square it so to undo that operation we take the square root of the area to get one side.

$\downarrow P = s + s + s + s$  or  $P = \sqrt{A} + \sqrt{A} + \sqrt{A} + \sqrt{A}$  so  $P = 4\sqrt{A}$


5. Express the surface area of a cube as a function of its volume.

$$SA = 6s^2$$

$$V = s^3$$

$$\sqrt[3]{V} = s$$

$$SA(V) = 6\sqrt[3]{V^2}$$



Surface Area - the area around the object.

Volume - how much the object holds

Same concept as the square before, given the volume find the SA. We have to negate the volume to one side. Since  $SA = 6s^2$  we square the side we got after negating the volume, then multiply 6.

6. Express the area of an equilateral triangle as a function of the length of a side.

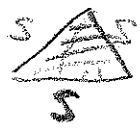
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}sh$$

$$A = \frac{1}{2}s\left(\frac{s\sqrt{3}}{2}\right)$$

$$A = \frac{s^2\sqrt{3}}{4}$$

$$h = \frac{s\sqrt{3}}{2}$$



Area - space inside the figure

Given a side of an equilateral triangle, find its area. All sides are equal, so  $b = s \Rightarrow A = \frac{1}{2}sh$  as  $h^2 + \frac{1}{4}s^2 = s^2$  to find height.

$h = \frac{s\sqrt{3}}{2} \Rightarrow A = \frac{1}{2}s\left(\frac{s\sqrt{3}}{2}\right) = \frac{1}{4}(s^2\sqrt{3}) = \frac{s^2\sqrt{3}}{4}$



**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

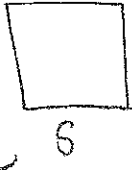
1. Express the perimeter of a square as a function of its area.

$P = 4s$   
 $A = s^2$   
 $\sqrt{A} = s$

$P = 4\sqrt{A}$

Define the variables.  
 Let  $P$  be the perimeter of the square,  $A$  be the Area and  $s$  be the length of one side.

So,  $P = 4s$       $A = s^2$   
 solve for  $s$  in the area formula, then substitute this value in the perimeter of a triangle formula.



2. Express the surface area of a cube as a function of its volume. *All sides equal!*

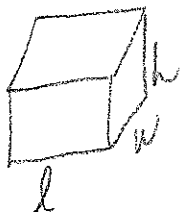
$SA = 2lw + 2wh + 2lh$   
 $V = lwh$

$h = \frac{V}{lw}$

$SA = 2lw + 2\left(\frac{V}{lw}\right) + 2lh$   
 $= 2lw + \frac{2V}{lw} + 2lh$

$SA = 2(lw) + \frac{2V}{lw} + 2l^2\frac{V}{lw}$

Write each formula.  
 Let  $SA$  be the surface area,  $l =$  length,  $w =$  width,  $h =$  height.  
 solve for  $h$  in the volume formula. (3) substitute this value into SA formula, then simplify.

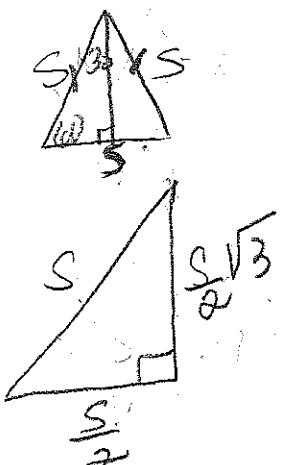


3. Express the area of an equilateral triangle as a function of the length of a side.

$A = \frac{1}{2} \left( \frac{s}{2} \cdot \frac{s\sqrt{3}}{2} \right)$   
 $= \frac{s}{8} \cdot \frac{s\sqrt{3}}{8} = \frac{s^2\sqrt{3}}{8}$

$A = \frac{s^2\sqrt{3}}{8}$

In an equilateral triangle, all sides are congruent.



**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader. (9<sup>th</sup>-grader)

4. Express the perimeter of a square as a function of its area.


① Draw a figure of a square

② Write the formula for the perimeter of a square, where  $P$  is the perimeter and  $s$  is the length of each side.  
 $P = 4s$

③ Write the formula for the area of the square, where  $A$  is the area and  $s$  is the length of each side:  $A = s^2$

④ Solve the Area formula for  $s$  by taking the square root of each side.  
 $A = s^2$   
 $\sqrt{A} = \sqrt{s^2}$   
 $\sqrt{A} = s$

⑤ Now, place the value of  $s$  in part 4 in the perimeter formula we wrote in part 2.  $P(A) = 4\sqrt{A}$



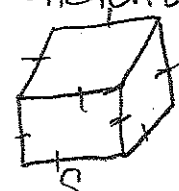
5. Express the surface area of a cube as a function of its volume.

① Sketch a cube. All edges are congruent.

② Write the formulas for the volume and surface area of a cube where  $S$  = surface area,  $V$  = volume and  $s$  represents the length of each edge.  
 $V = s^3$      $SA = 6s^2$

③ Since we must express the SA as a function of Volume, solve for  $s$  in the Volume formula, by taking the cube root of each side.  
 $\sqrt[3]{V} = \sqrt[3]{s^3}$   
 $\sqrt[3]{V} = s$

④ Substitute the value of  $s$  into the SA formula.  
 $SA = 6(\sqrt[3]{V})^2$



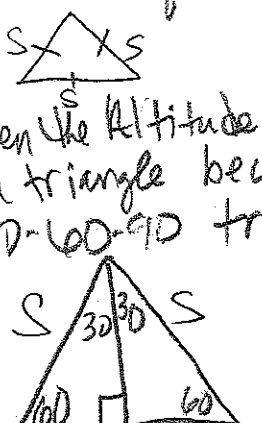
6. Express the area of an equilateral triangle as a function of the length of a side.

① Construct an equilateral  $\Delta$ . (All sides are congruent.)

② When the altitude is drawn, the triangle becomes 2 30-60-90 triangles.

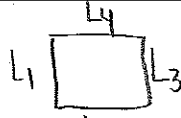
③ Use one of the triangles to find the height of the triangle, using the Pythagorean theorem.  
 $a^2 + (\frac{s}{2})^2 = s^2$   
 $h^2 + \frac{s^2}{4} = \frac{s^2}{1}$   
 $\sqrt{h^2 + \frac{s^2}{4}} = \sqrt{\frac{3s^2}{4}}$   
 $h = \frac{s\sqrt{3}}{2}$

④ Substitute the length of the base and the height into the formula:  $A = \frac{1}{2}bh$   
 $= \frac{1}{2}(s)(\frac{s\sqrt{3}}{2}) = \frac{s^2\sqrt{3}}{4}$



**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 miutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

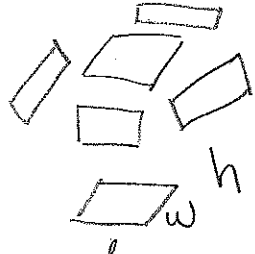
1. Express the perimeter of a square as a function of its area.



$P = \text{perimeter} = L_1 + L_2 + L_3 + L_4$   
 $P = 4L_1$   
 $A = \text{Area} = L_2^2$   
 $A = L_1^2$   
 $L_1 = L_2 = L_3 = L_4$   
 $L_1 = \sqrt{A}$   
 $P = 4\sqrt{A}$

since square

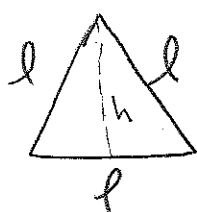
2. Express the surface area of a cube as a function of its volume.



$V = l = w = h$   
 $V = l^3$   
 $A = (wh)2 + 4(hl)$   
 $A = l^2 + 4(l^2)$   
 $A = 6l^2$   
 $l = \sqrt[3]{V}$   
 $A = 6(\sqrt[3]{V})^2$

Since cube  
 $l = w = h$

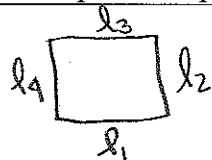
3. Express the area of an equilateral triangle as a function of the length of a side.



$h = \sqrt{l^2 - \left(\frac{l}{2}\right)^2}$   
 $h = \sqrt{l^2 \left(1 - \frac{1}{4}\right)}$   
 $A = \frac{b \times h}{2}$   
 where  $b = l$  and  $h$   
 $A = \frac{l \left( \sqrt{l^2 \left(1 - \frac{1}{4}\right)} \right)}{2} = \frac{l \left( l \sqrt{1 - \frac{1}{4}} \right)}{2}$   
 $A = \frac{l^2 \sqrt{1 - \frac{1}{4}}}{2}$

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

4. Express the perimeter of a square as a function of its area.



$$P = 4l$$

$$A = l^2$$

$P = \text{perimeter} = l_1 + l_2 + l_3 + l_4$  since we have a square  $l_1 = l_2 = l_3 = l_4$  and we can say

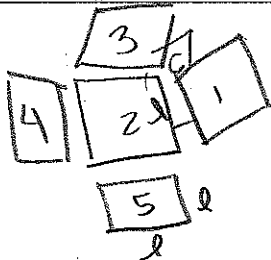
$$P = 4l$$

Now  $A = l \times l = l^2$  and from there  $l = \sqrt{A}$

Therefore

$$P = 4\sqrt{A}$$

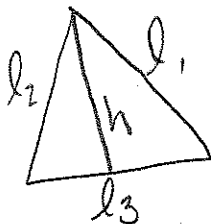
5. Express the surface area of a cube as a function of its volume.



A cube has 6 faces; then the surface area 6 times area of each face. Since each face is a square we have  $A = l^2$  then surface area of the cube  $6l^2$ . Now Volume =  $l^3$

from there  $l = \sqrt[3]{V}$  therefore  $A = 6(\sqrt[3]{V})^2$

6. Express the area of an equilateral triangle as a function of the length of a side.



We have an equilateral triangle meaning

$$l_1 = l_2 = l_3$$

Area of triangle  $\frac{b \times h}{2}$  where  $b = l_1$  and

$$h = \sqrt{l^2 - \left(\frac{l_3}{2}\right)^2} \leftarrow \text{from Pythagoras Theorem}$$

So we have  $A = \frac{1}{2} l^2 \left(\frac{l_3}{2}\right)$   $A = \frac{l \sqrt{l^2 - \left(\frac{l}{4}\right)^2}}{2}$   $A = \frac{l(l\sqrt{1 - \frac{1}{4}})}{2}$

$$A = \frac{l^2 \sqrt{3/4}}{2}$$

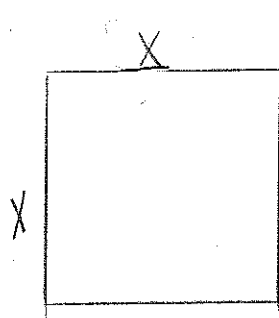
$$A = \frac{l^2 \frac{\sqrt{3}}{2}}{2}$$

$$A = \frac{l^2 \sqrt{3}}{4}$$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 miutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

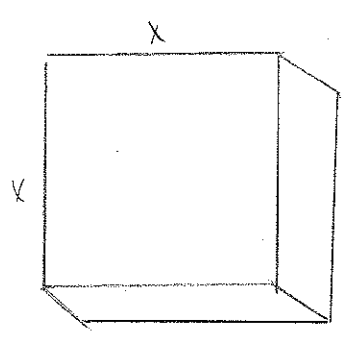
$$A(P(x)) = y$$



$P = 4x$        $A = x^2$

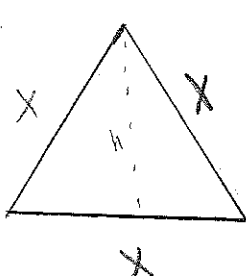
$P_{\text{erimeter}} = 4\sqrt{\text{area}}$   
 $P = 4\sqrt{x^2}$   
 $P = 4x$

2. Express the surface area of a cube as a function of its volume.



$\text{surface area} = 4x + 4x + 4x + 4x + 4x + 4x$   
 $= 6x$

3. Express the area of an equilateral triangle as a function of the length of a side.

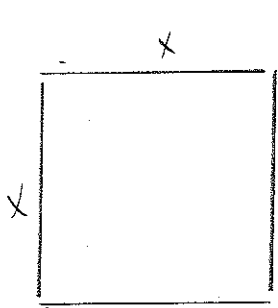


$A = \frac{1}{2}bh$  or  $\frac{1}{2}xh$

$A = \frac{x}{2}(h)$

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader.

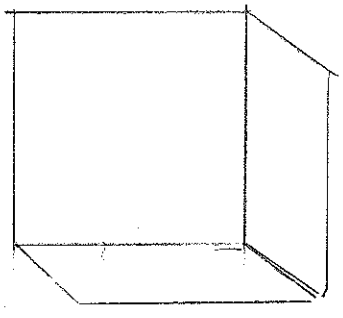
4. Express the perimeter of a square as a function of its area.



$A = x^2$        $P = 4x$   
 $P = 4\sqrt{x^2}$  or  $P = 4\sqrt{A}$

a square has 4 equal sides (x). We know the area (A) is equal to base times height which is  $x \cdot x$  or  $x^2$  in this case + Perimeter (P) is equal to  $x+x+x+x$  or  $4x$ . Therefore P is equal to 4 times the square root of  $x^2$  or  $P = 4\sqrt{x^2}$  /  $P = 4\sqrt{A}$

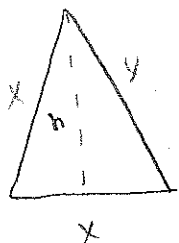
5. Express the surface area of a cube as a function of its volume.



Surface area =  $6x^2$   
 Volume =  $x^3$   
 Surface area =  $6(\sqrt[3]{x^3})^2$

if area =  $6x^2$  and volume =  $x^3$  we can take the cube root of  $x^3$  square it multiply 6 and get the area.  
 Surface area =  $6(\sqrt[3]{\text{volume}})^2$

6. Express the area of an equilateral triangle as a function of the length of a side.



$x^2 = (\frac{x}{2})^2 + h^2$   
 $\sqrt{x^2 = \frac{x^2}{4} + h^2}$   
 $x = \frac{x^2}{4} + h$   
 $2x = x + 2h$   
 $\frac{x}{2} = \frac{2h}{2}$        $h = \frac{x}{2}$

**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.

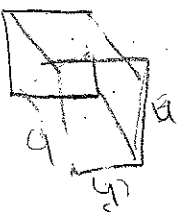
Area can be expressed as  $l \cdot w$ . Knowing the properties of a square it can be determined that the area is a square number. Take the square root of the perfect square and that gives you the sides. Perimeter eq

a square is  $4 \cdot s$ . so:  $\downarrow$

$$A = s^2 \quad P = \sqrt{A} \cdot 4$$

2. Express the surface area of a cube as a function of its volume.

let  $a$  be equal to the length of each side + each edge.



$V = l \cdot w \cdot h$


$V = a \cdot a \cdot a$

$V = a^3$

$SA = 6a^2$

$SA = 6 \sqrt[3]{V}$

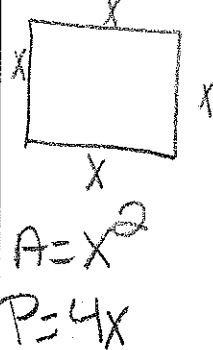
3. Express the area of an equilateral triangle as a function of the length of a side.



no clue

**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader. (9)

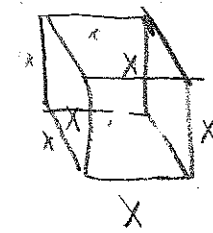
4. Express the perimeter of a square as a function of its area.



$A = x^2$   
 $P = 4x$

- All sides of a square are equal, let's call them  $x$
- We know that Area =  $x^2$   $A = x^2$
- Solve area in terms of  $x$  ( $x = \sqrt{A}$ )
- We also know that Perimeter =  $4x$
- Use substitution and you will get  $P = 4\sqrt{A}$

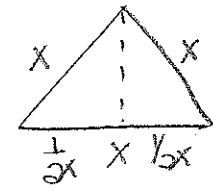
5. Express the surface area of a cube as a function of its volume.



Volume =  $l \cdot w \cdot h$   
or  
 $x \cdot x \cdot x = x^3$

- All sides of a cube are equal, let's call them  $x$ .
- Surface area is the area of all sides added.
- We have 6 sides, and we know  $A = x^2$ , so  
Surface area =  $6x^2$
- Using substitution, replace the  $x$  value in surface area with the value we found for volume. You get  $6(\sqrt[3]{V})^2$

6. Express the area of an equilateral triangle as a function of the length of a side.



Plug into area  
 $A = \frac{1}{2}(\frac{\sqrt{3}}{4}x)(x)$   
 $A = \frac{\sqrt{3}}{4}x^2$

- All sides are equal so call them  $x$ .
- Area of triangle =  $\frac{1}{2}bh$
- To find area we need to find height.
- Break triangle into half.
- Use Pythagorean theorem to find height.
- Plug into area formula.

$A = \frac{\sqrt{3}}{4}x^2$

Pythag. Theorem

$$(\frac{1}{2}x)^2 + h^2 = x^2$$

$$\frac{1}{4}x^2 + h^2 = x^2 \leftarrow \text{solve for } h$$

$$h^2 = x^2 - \frac{1}{4}x^2 \leftarrow \text{Common denom.}$$

$$h^2 = \frac{4}{4}x^2 - \frac{1}{4}x^2$$


$$h^2 = \frac{3}{4}x^2$$

$$h = \frac{\sqrt{3}}{2}x$$



**PART I:** Please work on this for 10 minutes your own, without discussing with your classmates. After 10 minutes, turn this sheet over and DO NOT WRITE FURTHER ON IT.

1. Express the perimeter of a square as a function of its area.




$P = 4s$   
 $A = s^2$

$P(A) = P(s^2)$

$P = 4(s^2)$

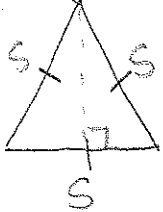
2. Express the surface area of a cube as a function of its volume.



$SA = 2s^2 + 2s^2 + 2s^2$   
 $= 6s^2$   
 $V = s^3$

$SA(V) = SA(s^3)$   
 $SA = 2s^3 + 2s^3 + 2s^3$   
 $SA = 6s^3$

3. Express the area of an equilateral triangle as a function of the length of a side.



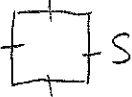
$A = \left(\frac{1}{2}s\right) \left(\sqrt{s^2 - \left(\frac{1}{2}s\right)^2}\right)$   
 $A(s) = \frac{1}{2}s \left(\sqrt{\left(\frac{3}{4}s\right)^2}\right)$   
 $= \frac{1}{4}s^2 \sqrt{3}$

$1 - \frac{1}{4} = \frac{3}{4}$

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**PART II:** After discussing the problems with your table, rewrite solutions. Write them so they will be clear and informative to an 8<sup>th</sup>-grader. ← don't do in 8<sup>th</sup> Grade! only functions done are  $f(x) = x + 3$ ;  $x = 2$

4. Express the perimeter of a square as a function of its area.



formula for area of a square:  $A = s^2$   
 formula for perimeter of a square:  $P = 4s$

$$P(A) =$$

$$= P(A = s^2)$$


$$= P(s^2)$$

$$P = 4(s^2)$$

$$P = 4(\sqrt{A})$$

$P = \text{perimeter}$   
 $A = \text{area}$   
 $s = \text{side length}$

5. Express the surface area of a cube as a function of its volume.



formula for volume of a cube:  $V = s^3$   
 formula for surface area of a cube:  $SA = 2s^2 + 2s^2 + 2s^2 = 6s^2$

$$SA(V) = SA(V = s^3)$$

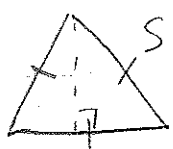
$$= SA(s^3)$$

$$= 6(s^3)^2$$

$$= 6(\sqrt[3]{V})^2$$

$SA = \text{surface Area}$   
 $V = \text{volume}$   
 $s = \text{side length}$

6. Express the area of an equilateral triangle as a function of the length of a side.



formula for area of a triangle:  $A = \frac{1}{2} b \cdot h$

$a^2 + b^2 = c^2$  ← Use Pythagorean Theorem to find height of triangle

$a^2 + (\frac{1}{2}s)^2 = s^2$

$a^2 = s^2 - (\frac{1}{2}s)^2$

$a = \sqrt{s^2 - (\frac{1}{2}s)^2}$

$a = \sqrt{s^2 - \frac{1}{4}s^2}$

$A = (\frac{1}{2}s)(\frac{1}{2}s\sqrt{3})$

$A = \frac{1}{4}s^2\sqrt{3}$

$a = \sqrt{\frac{3}{4}s^2}$  ←

$a = \frac{1}{2}s\sqrt{3}$

$s = \text{side length}$   
 $A = \text{area}$   
 $a, b, c = \text{Pythagorean Theorem}$