**Some ideas from high-school geometry**

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The following outline contains a summary of some central ideas from the 9th/10th grade geometry textbook currently used in East Baton Rouge Parish, *Geometry: Integration, Applications, and Connections*, Glencoe 1998. I have not attempted to describe the organization of the Glencoe course, but only isolate what are, from a mathematical point of view, the most significant geometric ideas. What is the purpose? First, anyone studying geometry in order to prepare for teaching high school should be in complete control of everything mentioned below. Future teachers should also have a thorough understanding of how this knowledge fits into geometry conceived more generally as a mathematical field of knowledge and how this fits into the high-school curriculum.

**I.** High-school vocabulary: **point, line, angle, measure of an angle, right angle, supplement of an angle, vertical angles, perpendicular lines, parallel lines, triangle, quadrilateral, parallelogram, rhombus, rectangle, square, polygon, congruent segments, congruent angles, congruent triangles, similar triangles, circle, diameter, chord, arc, tangent**

**II.** High-school constructions:
- **A.** Given a segment, construct its perpendicular bisector.
- **B.** Given an angle, construct the bisector of that angle.
- **C.** Given a line and a point, construct the line through the point perpendicular to the given line.
- **D.** Given a line and a point not on the line, construct the line through the point parallel to the given line.

**III.** Basic high-school facts. The following include some *axioms* (*i.e.*, unproved basic assumptions) of classical geometry (such as SAS and the parallel postulate) as well as some *theorems* (*i.e.*, deduced truths) such as ASA, SSS and the criterion for parallelism in **E**. The facts listed here do not include every fundamental truth that one might use in a geometric argument. For example, "Two points determine a line" is certainly a fact that we appeal to over and over again, but it is so well-known that I omitted it from the list, assuming that no one needs to be reminded. Most people use these facts without questioning them; they are "pragmatic" axioms in high-school geometry.
- **A.** Supplements of congruent angles are congruent; vertical angles are congruent.
- **B.** SAS, ASA and SSS criteria for triangle congruence.
- **C.** AAA criterion for triangle similarity.
- **D.** *The parallel postulate*. If parallel lines are cut by a transversal, then corresponding angles are congruent.
- **E.** If two lines are both perpendicular to a third line, then the two lines are parallel.

**IV.** Some important high-school geometry theorems. Each of the following is proved in Glencoe *Geometry*, or the proof is assigned as an exercise.
- **A.** If two lines are cut by a transversal and corresponding angles are congruent, then the two lines are parallel.
- **B.** The sum of the measures of the angles of a triangle is $180°$.
- **C.** The sum of the measures of the interior angles of an $n$-gon (*i.e.*, a polygon with $n$ sides) is $(n-2)180°$.

**D.** *Parallelogram theorem*. For any quadrilateral, the following are equivalent:

*i.* Opposite sides are parallel;

*ii.* The diagonals bisect one another;

*iii.* Opposite sides are congruent;

*iv.* Opposite angles are congruent.

**E.** *Rhombus theorem*: The diagonals of a parallelogram are perpendicular if and only if all four sides sides are congruent. *Rectangle theorem*: The diagonals of a parallelogram are congruent if and only if all adjacent sides are perpendicular. *Note*: there is a quadrilateral that is neither a rhombus nor a rectangle, but whose diagonals are both perpendicular and congruent to one another.

**F.** *The Pythagorean Theorem and its converse*: In a right triangle, the sum of the squares of the legs equals the square of the hypotenuse. Conversely, if the sum of the squares of two sides of a triangle equals the square of the remaining side, the triangle is right.

**G.** The perpendicular bisector of any chord of a circle passes through the circle's center.

**H.** Let $P$ be a point on a circle $\gamma$. A line through $P$ is tangent to $\gamma$ if and only if it is perpendicular to the diameter through $P$.

**I.** If $A$, $B$ and $C$ are points on a circle, then the measure of angle $ABC$ is half the measure of the intercepted arc.

**V.** Some additional theorems from Glencoe *Geometry*:

**A.** An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides. (Prove this by constructing a line through a vertex parallel to the bisector; consider similar triangles.)

**B.** *The Law of Sines*: In triangle $ABC$, with sides $a$, $b$ and $c$ opposite angles $A$, $B$ and $C$ respectively:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

**C.** *The Law of Cosines*: In triangle $ABC$, with sides $a$, $b$ and $c$ opposite angles $A$, $B$ and $C$ respectively:

$$c^2 = a^2 + b^2 - 2ab\cos C.$$

**D.** If lines intersect in the interior of a circle, then the measure of an angle is one half of the sum of the measures of the arcs intercepted by the angle and its vertical angle. (This is a more general version of IV.I.)

**E.** If $A$, $B$, $C$ and $D$ are 4 points on a circle, and $AB$ meets $CD$ at a point $E$, then $AE \cdot EB = CE \cdot ED$. (First prove the special case when $E$ is in the interior.)