18.01A Exam 2 Review

1. Find the partial fraction decompositions for:

a) $\dfrac{5x}{(x-1)(x^2+4)}$

b) $\dfrac{x^3}{x^3-x}$

2. Evaluate the following integrals through substitutions and/or partial fractions:

a) $\displaystyle\int \frac{5x}{(x-1)(x^2+4)}\,dx$

b) $\displaystyle\int_0^1 \frac{x^3}{9+x^2}\,dx$

c) $\displaystyle\int \frac{x^3(x^2-9)^{\frac{3}{2}}}{9}\,dx$

d) $\displaystyle\int_{-2}^0 \frac{1}{x^2+4x+8}\,dx$

3. Evaluate the following integrals by using integration by parts:

a) $\displaystyle\int_0^1 x^3 \ln x\,dx$

b) $\displaystyle\int e^{-x}\cos x\,dx$

c) $\displaystyle\int x\cos^{-1} x\,dx$

4. Evaluate the following integrals with trigonometric identities and reduction tech-

niques:

a) $\displaystyle\int \sin^3 x \cos^2 x \, dx$

b) $\displaystyle\int \cos^4 x \sin^2 x \, dx$

c) $\displaystyle\int_0^{\pi} \cos^2 x \, dx$

5. Determine whether or not the following improper integrals of the first kind (infinite domain) converge:

a) $\displaystyle\int_1^{\infty} \frac{1}{x(x+3)} \, dx$

b) $\displaystyle\int_0^{\infty} x\sqrt{e^{-x}} \, dx$

c) $\displaystyle\int_1^{\infty} \frac{x}{x^2+3x+2} \, dx$

d) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt[3]{(x+2)(x+7)}} \, dx$

6. Determine whether or not the following improper integrals of the second kind (infinite range) converge:

a) $\displaystyle\int_2^{3} \frac{x}{x^2-2x+1} \, dx$

b) $\displaystyle\int_0^{1} \frac{x+2}{(x-1)^2} \, dx$

c) $\displaystyle\int_0^{2} \frac{x}{x^2+3x+2} \, dx$

d) $\displaystyle\int_0^{1} \frac{\ln x}{\sqrt{x}} \, dx$

7. Consider the curve $y = \frac{1}{\sqrt{x}\ln x}$ from $2 \leq x \leq \infty$, and rotate it around the x-axis. It is clearly infinitely long, but the following exercises ask whether other physical aspects

are finite or not (the fact that this is even in question should strike you as remarkable, as an infinitely long vessel is potentially finite in volume and/or surface area!)

a) Does $\displaystyle\int_2^\infty \frac{1}{\sqrt{x}\ln x}\,dx$ converge?

b) Is the volume finite?

c) Is the surface area finite?

8. Determine whether or not the following series converge:

a) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2-3}$

b) $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(\ln r)^3}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\frac{1}{n}}{\sqrt{n^2+1}}$

9. Show that each of the following series converges and find an upper bound for its value:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

b) $\displaystyle\sum_{r=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$

Fun Fact. The number of seconds in a year is

$$31536000 \approx \pi \cdot 10^7.$$