18.01A Exam 3 Review

<u>1.</u> Suppose that a brand of gum contains many different colors, which are distributed as:

Color	Red	Green	Blue	Yellow	Purple	Orange
No. of pieces	2	4	5	1	3	2

   a) Calculate the mean for the number of pieces per color.
   b) Calculate the standard deviation.

<u>2.</u> The discrete random variable x takes on values from 0 to ∞ according to the probability function

$$P(x = n) := \frac{2}{3} \cdot \left(\frac{1}{3}\right)^n.$$

   a) Show that this defines a probability distribution (i.e. total probability $= 1$).
   b) Calculate the expected value of x.
   c) Calculate the standard deviation of x.

<u>3.</u> In an average week, a person sees 3 dogs on the street.
   a) What is the probability that the person sees at most 2 dogs in a week?
   b) What is the probability that the person sees exactly 10 dogs in 4 weeks?

<u>4.</u> Define a probability density function by

$$f(x) = Ax, \qquad 0 \leq x \leq 2.$$

   a) Find the value of A.
   b) Find the mean of x.
   c) Find the standard deviation of x.

<u>5.</u> Define a probability density function by

$$f(x) := A \cdot \frac{\tan^{-1} x}{x^2}.$$

   a) Why is this a valid probability density function?
   b) Find the value of A.

<u>6.</u> On average I wait 20 minutes for the bus to pass.

a) What is the probability that I will have to wait for 40 minutes or longer?

b) What is the probability that I wait less than 10 minutes twice in a row?

7. Define a probability density function by

$$f(x) := \frac{A}{x^4}.$$

a) Find the value of A.

b) Find the expected value of x.

c) Calculate the variance of x.

d) Determine the associated probability distribution $F(x)$.

8. For each $r \geq 0$, define

$$f_r(x) := \frac{A_r x^2}{(3 + x^3)^r}.$$

a) For which values of r is this a probability density? When it is, what is A_r?

b) For which r is the mean finite?

c) For which r is the variance finite?

9. The Intelligence Quotient (IQ) is designed to have a mean of 100 points, with a standard deviation of 15.

a) What is the cut-off for MENSA, which permits the top 2% of IQ scores?

b) What percentage of people have an IQ between 85 and 115?

c) What percentage of people have an IQ between 80 and 120?

10. In a certain state, cars sell at an average price of $17000, with a standard deviation of $4000. However, a dealer sells 100 cars at an average price of $17,600$. What is the probability of this happening by chance?

11. A poll of 3600 people finds that 63% of them believe that most statistics are just made up to provide extra practice in math classes.

a) What is the margin of error in this poll with a confidence level of 95%?

b) How many people must be polled to achieve an error of 1% with 97% confidence?

12. I believe that 72% of MIT students work hard in their classes. However, a poll of 100 students reveals that 78% of students claim to be working diligently. With what confidence level was my initial estimate incorrect?

Fun Fact. There are two people living in Cambridge who have exactly the same number of hairs on their heads!