18.02A Exam 2 Review

1. Consider the set of all lines that pass through the point $(3, 1)$ and intercept both the x and y axes in the first quadrant (i.e., negative slope). Let X and Y denote the x and y intercepts, respectively.
   a) Minimize $X + Y$.
   b) Minimize XY.

2. Classify the critical points of the function

$$f(x, y) = x^3 + 2y^2 - 4xy.$$

3. Find the closest point to the origin on the surface

$$x^2 + 2y + z = 3.$$

4. Let $f(x, y, z) = 3x^2 e^{yz}$. Calculate df/dt along the parameterization $x = t^2,\ y = \sec t,\ z = \frac{2}{t}$.

5. Define $w(x, y, t) = xe^t + ty^2$, where the variables also satisfy $t^2 = \sin(xy)$. Find the constrained partial derivative $\left(\frac{\partial w}{\partial t}\right)_x$ by using:
   a) Chain Rule
   b) Differentials

6. Consider the function $w = \sqrt{x^2 + y^2}$ with the constraint $u = y/x$.
   a) Use the method of differentials to calculate $\left(\frac{\partial w}{\partial x}\right)_u$.
   b) Provide a geometric explanation for the answer in a).

7. Evaluate the double integral

$$\int_2^3 \int_y^{y^2} 2x \, dx \, dy.$$

8. Switch the order of integration in order to evaluate the integral

$$\int_0^2 \int_0^{4-x^2} 2x \cos(8y - y^2) \, dy \, dx.$$

9. The region between the curves $y = x$ and $y = x^2$ has a mass density $\delta = \sqrt{x}$.

a) Calculate the center of mass.

b) Calculate the moments of inertia.

10. A region is bounded on the right by $x = a$, and on all other sides by "inverted circles," which are given by

$$y = \pm \left(a - \sqrt{a^2 - x^2} \right).$$

a) Sketch the region.

b) Show that the left boundary of the region is described by the polar equation $r = 2a \sin \theta$.

c) Find the center of mass.

11. Evaluate the following integrals by converting them to polar coordinates:

$$\text{a)} \quad \int_0^1 \int_{x^2}^{y} x^2 + y^2 \, dy \, dx,$$

$$\text{b)} \quad \int_0^3 \int_0^{\sqrt{9-x^2}} 3y \, dy \, dx.$$

12. A disc rolls without slipping up an incline with slope 1/2 in the x-y plane. At time $t = 0$, the point P on the edge of the disc is at the origin. Describe the path of P with a parametric equation.

Fun Fact. It's impossible to smoothly comb a "Hairy Sphere!"