18.786 Problem Set 3 - Spring 2008

Due Thursday, Mar. 6 at 1:00

1. (a) Exercise 4 on page 29 of Janusz.
   (b) Exercise 5 on page 30 of Janusz.
2. Exercise 6 on page 32 of Janusz. Take special note of what this implies for ramification in the Gaussian integers ($d = -1$).
3. Determine the set $\mu := \{\alpha \in \mathbb{Q}[\sqrt{-d}] \ : \ d \in \mathbb{N}, \alpha = \text{cplx. root of } 1\}$. Note that this is **not** the set of multiplicative units in $\mathbb{Q}[\sqrt{d}]$. (*Hint: μ is a finite set – look at extension dimensions*).
4. A *biquadratic field* is a number field of degree 4 of the form $\mathbb{Q}[\sqrt{d_1}, \sqrt{d_2}]$ for distinct $d_i \in \mathbb{Z}$.
   (a) Determine which of the cyclotomic fields $\mathbb{Q}[\zeta_p]$ are biquadratic.
   (b) Characterize the possible ramification degrees for primes in a biquadratic extension. In particular, there is only one prime $\mathfrak{p} \subset \mathbb{Z}$ that can have ramification degree 4.