**MATH 18.02A** - Feb. 12 Recitation

**1.** Evaluate the integral

$$\int_0^2 \int_1^2 \int_{2y-2}^2 2xe^{z^2 x}\, dz\, dy\, dx$$

by switching the order of integration to $dy\, dz\, dx$. (*Ans:* $\frac{e^8}{8} - 1$)

**2. a)** Compute the moment of inertia $I_z$ for the solid bounded by $z = 0$, $z = x^2 + y^2$ and $x^2 + y^2 = a^2$ (assume constant density $\delta$). (*Ans:* $\frac{\pi \rho a^6}{3}$)
**b)** Compute the volume of a prism of height $b$ and a circular base of radius $a$. *Hint: Place the base on the $(x, y)$ plane, and the peak at $(0, 0, b)$.* (*Ans:* $\frac{\pi a^2 b}{3}$)

**Gravitation Facts:**

- The gravitational force of a point mass $m_2$ acting on a mass $m_1$ is given by
$$\frac{Gm_1 m_2}{|\mathbf{r}|^2}\, \hat{\mathbf{r}},$$
where $\mathbf{r}$ is a vector pointing from $m_1$ to $m_2$, and $\hat{\mathbf{r}}$ is its unit direction vector.
- A spherical shell of uniform density may be viewed as a point mass located at its center when calculating the gravitational force on a mass outside the shell.
- A mass located entirely inside a spherical shell of uniform density experiences no net gravitational force (in this case, the shell must be hollow).

**3.** A sphere of radius $a$ and density $\delta$ is placed in the center of a spherical shell of density $\frac{\delta}{5}$, whose inner radius is $2a$, and whose outer radius is $3a$. A point of mass $m$ is placed outside of the shell, at a distance $r$ from the shared center of the spheres.
**a)** What is the total gravitational force felt by the inner sphere? (*Ans:* $\frac{4\pi G m a^3}{3r^2}$)
**b)** What is the total gravitational force felt by the point mass? (*Ans:* $\frac{8\pi G m a^3}{3r^2}$)

**4. a)** A hemisphere with radius $a$ and uniform density $\delta$ is placed with its center at the origin. Find the force it exerts on a mass of $m$ placed at the origin. (*Ans:* $\pi m a G \delta\, \hat{\mathbf{k}}$.)
**b)** Now suppose that the hemisphere has density $\delta(x, y, z) = x^2 + y^2$ and find the resulting force. (*Ans:* $\frac{\pi m a^3 G \delta}{6}\, \hat{\mathbf{k}}$.)