MATH 18.02A - Feb. 14 Recitation

1. Sketch the following vector fields:
a) $\mathbf{F}(x, y) = y\,\hat{\mathbf{i}} - x\,\hat{\mathbf{j}}$. (*Ans: Clockwise rotation*)
b) $\mathbf{G}(x, y) = \sin x\,\hat{\mathbf{i}}$.
c) $\mathbf{H}(x, y) = \hat{\mathbf{i}} + y(1-y)\,\hat{\mathbf{j}}$.

2. a) Let $\mathbf{c}$ be the clockwise, circular path of radius 2 between the points $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, \sqrt{2})$, and define a vector field by $\mathbf{F}(x, y) = y\,\hat{\mathbf{i}} - x\,\hat{\mathbf{i}}$. Calculate $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{r}$. (*Ans:* 2π)
b) Explain the answer to **a)** geometrically.
c) If $\mathbf{c}(t) = (1, t^2, t)$ for $0 \leq t \leq 2$, calculate the line integral

$$\int_{\mathbf{c}} \sin y\, dx + \frac{z^2}{4}\, dy + 2ze^y\, dz. \qquad \big(\textit{Ans: } 1 + e^4\big)$$

3. Define the vector field $\mathbf{F}(x, y) = \frac{1}{x}\,\hat{\mathbf{i}} + \frac{1}{y}\,\hat{\mathbf{j}}$, and calculate the line integral for the following paths between $(1, 1)$ and $(2, 4)$:
a) $\mathbf{c}(t) = (1 + t, 1 + 3t)$ for $0 \leq t \leq 1$. (*Ans:* $3 \ln 2$)
b) $\mathbf{c}(t) = (t, t^2)$ for $1 \leq t \leq 2$. (*Ans:* $3 \ln 2$)