MATH 18.02A - Feb. 20 Recitation

1. Let $\mathbf{F} = \vec{\nabla}(x^2y + \sin\frac{\pi y}{2})$. The Fundamental Theorem of Line Integrals will help in the following calculations.

a) Use only the endpoints to evaluate

$$\int_{(0,1)}^{(2,2)} \mathbf{F}\cdot d\mathbf{r}. \qquad (Ans: 7)$$

b) Now write out $\mathbf{F}$ explicitly, and define the path $\mathbf{c}(t) = (1+t, 2-\cos(2\pi t))$ for $0 \le t \le 1$. Use path independence (i.e., choose a simpler path!) to evaluate $\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{r}$. (*Ans:* 3)

2. Calculate the curl of the following vector fields.

a) $\mathbf{F} = y^2\,\hat{\mathbf{i}} + x^2\,\hat{\mathbf{j}}$. (*Ans:* $2x - 2y$)
b) $\mathbf{F} = \sec^2 xy\,\hat{\mathbf{i}} + \tan x\,\hat{\mathbf{j}}$. (*Ans:* 0)
c) $\mathbf{F} = -2y\sin^2 x\,\hat{\mathbf{i}} + (\sin x\cos x + 3e^y)\,\hat{\mathbf{j}}$. (*Ans:* 1)

3. Identify which of the following fields are conservative. For those that are, find a potential function $f(x,y)$. Practice using both the algebraic and integration methods.

a) $\mathbf{F} = x^2\,\hat{\mathbf{i}} + y^2\,\hat{\mathbf{j}}$. (*Ans: Yes;* $f(x,y) = \frac{x^3}{3} + \frac{y^3}{3}$)
b) $\mathbf{F} = y^2\,\hat{\mathbf{i}} + x^2\,\hat{\mathbf{j}}$. (*Ans: No*)
c) $\mathbf{F} = ye^{e^x+x}\,\hat{\mathbf{i}} + e^{e^x}\,\hat{\mathbf{j}}$. (*Ans: Yes;* $f(x,y) = ye^{e^x}$)
d) $\mathbf{F} = (-2x\sin(1+x^2) + \sin y^2)\,\hat{\mathbf{i}} + 2xy\cos y^2\,\hat{\mathbf{j}}$. (*Ans: Yes;* $f(x,y) = \cos(1+x^2) + x\sin y^2$)
e) $\mathbf{F} = (\tan^{-1} y + 2)\,\hat{\mathbf{i}} - \frac{x}{1+y^2}\,\hat{\mathbf{j}}$. (*Ans: No*)

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