MATH 18.02A - Feb. 26 Recitation

1. a) Show that $\oint_{\mathbf{c}} -x^4y^3\,dx + 2xy^2\,dy$ is always positive for a simple, closed curve $\mathbf{c}$.
b) Suppose that $\mathbf{c}$ lies in the second quadrant ($x \le 0, y \ge 0$). What is the sign of $\oint_{\mathbf{c}} xy^2\,dx - x^4y\,dy$? (*Ans: positive*)

2. a) Use Green's Theorem to find the area between the curves $y = x^2$ and $x = y^3$. (*Ans:* $\frac{5}{12}$)
b) Find the area bounded by the x-axis and one arch of the cycloid

$$x = \theta - \sin\theta, \qquad y = 1 - \cos\theta, \qquad 0 \le \theta \le 2\pi. \qquad (Ans: 4\pi)$$

Remark. Green's Theorem can simplify the calculation of the area bounded by a simple, closed curve. In particular,

$$\iint_R 1\,dA = \oint_{\partial R} -y\,dx = \oint_{\partial R} x\,dy = \frac{1}{2}\oint_{\partial R} -y\,dx + x\,dy.$$

3. a) Find the path $\mathbf{c}$ that maximizes

$$\int_{\mathbf{c}} x^2y\,dx + (x - xy^2)\,dy.$$

(*Hint: Use Green's Theorem*) (*Ans: Circle of radius 1*)
b) Calculate the maximum value. (*Ans:* $\frac{\pi}{2}$)