MATH 18.02A - Feb. 28 Recitation

1. Let $\mathbf{c}$ be a positively oriented curve that begins somewhere along the positive y-axis and ends at $(3, 0)$, staying in the first quadrant at all times. Calculate the flux of $\mathbf{F} = x^4 y\,\hat{\mathbf{\imath}} + (-2x^3y^2 - 2x\,\hat{\mathbf{\jmath}}$ across $\mathbf{c}$. (*Ans:* -9)
Hint: Use Green's Theorem on the closed loop consisting of $\mathbf{c}$ *and segments of the* x*- and* y*-axes.*

2. Consider the field $\mathbf{F} = \frac{x}{r^2}\,\hat{\mathbf{\imath}} + \frac{y}{r^2}\,\hat{\mathbf{\jmath}}$, and suppose that $\mathbf{c}$ is a simple closed curve that surrounds the origin. Let $\mathbf{c}_1$ be a large circle that completely contains $\mathbf{c}$, and use Green's Theorem on the region between $\mathbf{c}$ and $\mathbf{c}_1$ to calculate the flux across $\mathbf{c}$. (*Ans:* 2π)

Remark. If $\mathbf{F} = r^n(x\,\hat{\mathbf{\imath}} + y\,\hat{\mathbf{\jmath}})$, then the divergence is non-zero for all $n \neq -2$. When $n = -2$, the divergence is zero everywhere except at the origin, where there is a source of rate 2π (compare with the calculations in problem **2**).