MATH 18.02A - Mar. 7 Recitation

1. Let $\mathbf{c}_1$ be the triangular path through the points in order $(2,0,3),(2,2,3)$ and $(0,3,3)$, and let $\mathbf{c}_0$ be the path through $(2,0,0),(2,2,0)$ and $(0,3,0)$. Find the difference between the line integrals $\oint_{\mathbf{c}_1} \mathbf{F}\cdot d\mathbf{r}$ and $\oint_{\mathbf{c}_0} \mathbf{F}\cdot d\mathbf{r}$ for the field

$$\mathbf{F} = \left(x^4 \cos\sqrt{1+x^3} + zy^2\right)\hat{\mathbf{\imath}} + (2xyz - 2x)\,\hat{\mathbf{\jmath}} + (xy^2 - 3^{z^2+1})\,\hat{\mathbf{k}}.$$

Hint: Using Stokes' Theorem, it isn't necessary to actually compute any line integrals. (*Ans:* 0)

2. a) Let S be the triangle bounded by $(0,0,0),(0,2,0)$ and $(0,0,1)$ with orientation $\hat{\mathbf{n}} = \hat{\mathbf{\imath}}$. Calculate the flux of $\mathbf{F} = (xyz + 2y)\,\hat{\mathbf{\imath}} + (-\sin(x-y) + z^3y^2)\,\hat{\mathbf{k}}$. (*Ans:* $\frac{4}{3}$)

b) Calculate the flux $\iint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dA$ of $\mathbf{F} = e^{-y}\,\hat{\mathbf{\imath}} + z\,\hat{\mathbf{\jmath}} + (x^2-2)\,\hat{\mathbf{k}}$ through the surface S in the first quadrant that is bounded by $x^2+y+z=2$ and oriented so that the normals point away from the origin.

Hint: Write the surface as a graph $z = f(x,y)$ and recall that in this case the normal differential is $\hat{\boldsymbol{n}}\,dA = d\mathbf{S} = -f_x\,\hat{\boldsymbol{\imath}} - f_y\,\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}$. $\left(Ans: \frac{-4\sqrt{2}}{3} - 1\right)$

c) Let S be the lower hemisphere ($z \le 0$) of radius a with normals pointing towards to the origin. Calculate the flux of $\mathbf{F} = \hat{\mathbf{k}}$.

Hint: The outward normal differential for points (x,y,z) on a sphere of radius a is always $\hat{\boldsymbol{n}}\,dA = a^2\sin\phi\left(\frac{x\,\hat{\imath}+y\,\hat{\jmath}+z\,\hat{k}}{a}\right)$, where ϕ is the spherical coordinate of the point (x,y,z). (*Ans:* πa^2)