MATH 18.02A - Mar. 12 Recitation

1. A surface S is defined to be the positive $(z \geq 0)$ graph of a function $z = f(x, y)$, and the intersection with the (x, y) plane is a closed curve that bounds with area A. Calculate $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}} \, dA$ for the field $\mathbf{F} = (3y^4 + 2x)\,\hat{\mathbf{\imath}} + (\cos z - e^2 x - 2y)\,\hat{\mathbf{\jmath}} + 3\,\hat{\mathbf{k}}$. *Hint: Use the Divergence Theorem.* (*Ans:* $3A$)

2. a) Let R be the region bounded by $z^2 = x^2 + y^2$ and $z = \sqrt{1 - x^2 - y^2}$. Calculate the total flux outward through the boundary of R of the field $\mathbf{F} = -x^3\,\hat{\mathbf{\imath}} - y^3\,\hat{\mathbf{\jmath}}$. $\left(Ans : \frac{\pi}{\sqrt{2}} - \frac{4\pi}{5}\right)$

b) Calculate the flux $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}} \, dA$ of $\mathbf{F} = z\,\hat{\mathbf{\imath}} - y\,\hat{\mathbf{\jmath}} + (x - z)\,\hat{\mathbf{k}}$ through the surface S in the first quadrant that is bounded by $2x + y + z = 2$ and oriented so that the normals point away from the origin. $\left(Ans : \frac{-5}{3}\right)$

c) Let S be the lower hemisphere $(z \leq 0)$ of radius a with normals pointing towards to the origin. Calculate the flux of $\mathbf{F} = \hat{\mathbf{k}}$. (*Ans:* πa^2)