18.02A Exam 1 - Spring 2007

Friday, Feb. 23

Name: ______________________________

Directions: Answer all questions (pages are printed front and back). Read the problem statements *carefully,* and show your work for full credit. No books, notes, calculators, or aids of any kind are allowed.

	Score
Problem 1 (*20 pts*)	
Problem 2 (*20 pts*)	
Problem 3 (*20 pts*)	
Problem 4 (*20 pts*)	
Problem 5 (*20 pts*)	
Total (*100 pts*)	

Problem 1. (*Short answer; 5 pts each*) You are not required to show detailed work for these questions.

a) Suppose that $\mathbf{F} = \vec{\nabla} f(x, y)$, and that $\mathbf{c}$ is a level curve of f (so that $f(x, y)$ has a constant value along $\mathbf{c}$). Find $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{r}$ (*Hint: No calculation is necessary*).

b) A triple integral is to be evaluated using the change of variables $u = x^2 + z, v = x \sin y$, and $w = y - 2z$. Fill in the entries in the blank matrix:

$$\iiint_{R_{xyz}} f(x, y, z)\, dV = \iiint_{R_{uvw}} g(u, v, w) \begin{vmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{vmatrix}^{-1} du dv dw.$$

c) Let $\mathbf{F}$ be a constant field $a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$. Calculate

$$\int_{(x_0, y_0, z_0)}^{(x_1, y_1, z_1)} \mathbf{F} \cdot d\mathbf{r}.$$

d) At (x, y), the vector field $\mathbf{F}$ is defined by rotating the radially inward unit vector in the clockwise direction by an angle of $\frac{\pi}{2.}$ Write down the explicit equations for $\mathbf{F}$.

Problem 2. (*20 pts*) Evaluate the integral

$$\iint_R 2(x-3y)^2(x-y)\,dA,$$

where R is the triangle with vertices $(0,0), (1,1)$, and $(3,1)$.
(*Hint: Make the substitutions* $u=-x+3y, v=x-y$.)

Problem 3. (*20 pts*) Let R be the region bounded by the surface $z = 4 - x^2 - y^2$ and the (x, y) plane. Find the total mass if the density function is $\delta(x, y, z) = \sqrt{x^2 + y^2}$.

Problem 4. (*20 pts: 5+10+5*) Consider the vector field

$$\mathbf{F} = (6x^5y^2 + \cos x - 3e^y)\,\hat{\mathbf{\imath}} + (ax^6y - 3xe^y)\,\hat{\mathbf{\jmath}}.$$

a) Find the value(s) of a such that $\mathbf{F}$ is conservative.

b) For the value(s) of a determined in part a), find the potential function(s).

c) Find the work done by $\mathbf{F}$ along the path $\mathbf{c}(t) = (1 - t^2, t)$ for $1 \le t \le 2$.

Problem 5. (*20 pts: 15+5*)
a) One of the following fields is not conservative:

$$\mathbf{F} = 2x^2\,\hat{\mathbf{j}},$$
$$\mathbf{G} = (3x^2y - \sin x)\,\hat{\mathbf{i}} + x^3\,\hat{\mathbf{j}}.$$

Determine which one it is, and verify this by finding two different paths with unequal line integrals.

b) Suppose that $\mathbf{F} = (g(x) + h(y))\,\hat{\mathbf{j}}$ for some differentiable functions g and h. If $\mathbf{F}$ is known to be conservative, then what can be concluded about $g(x)$ and $h(y)$? (*Hint: There is a very strong restriction on at least one of these functions*)