18.02A Practice Test – IAP 2007 (Solutions to be posted 10am Jan.18)

Problem 1.

a) Write down in $xy-$coordinates the vector field $\mathbf{F}$ whose vector at (x, y) is obtained by rotating $90°$ counterclockwise the radially-outward-pointing unit vector at (x, y).

b) Let $\mathbf{F}$ be the field in part (a). Let C_1 be the line segment running from (1,1) to (2,2), and C_2 the positively-oriented circle of radius a centered on the origin. Using intuition, give the value of the following (short answer; no calculation required):

i) $\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$, ii) $\displaystyle\oint_{C_2} \mathbf{F} \cdot d\mathbf{r}$, iii) flux of $\mathbf{F}$ across C_1, iv) flux of $\mathbf{F}$ across C_2.

Problem 2.

Let $\mathbf{F} = \nabla f = \text{grad} f$, where $f(x, y) = x^2 + 4y^2$.

a) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a curve running from (1,1) to (2,2).

b) Find the locus of all points (x, y) in the plane such that $\displaystyle\int_{(1,1)}^{(x,y)} \mathbf{F} \cdot d\mathbf{r} = 0$.

Problem 3.

Let $\mathbf{F} = y(ax + y)\mathbf{i} + (3x^2 + bxy + y^3)\mathbf{j}$, where a, b are constants.

a) Prove: if $\mathbf{F}$ is conservative, then $a = 6$, $b = 2$. (Use these values in part b).

b) Using a systematic method (show work), find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$.

Problem 4.

Let C be the portion of the parabola $y = 1 - x^2$ lying over the $x-$axis, oriented in the direction of decreasing x. Taking

$$\mathbf{F} = (6xy^5)\mathbf{i} + (1 + x^2y - y^6)\mathbf{j},$$

a) set up an integral in x alone that represents the flux of $\mathbf{F}$ over C. (Give integrand and limits, but do not evaluate);

b) calculate the flux of $\mathbf{F}$ over C by using Green's Theorem in the normal form. (Note that C is not closed).

Problem 5.

Show that the value of $\displaystyle\oint_C (y^2 - 2y)\ dx + 2xy\ dy$ around a positively oriented circle C depends only on the size of the circle, and not on its position.

Problem 6.

Consider the integral $\displaystyle\int\int_R (x + y)^4(3x - y)^4 dxdy$, where R is the triangle with vertices at $x = -1$ and $x = 3$ on the $x-$axis, and $y = 3$ on the $y-$axis.

Let $u = x + y$ and $v = 3x - y$. Express the double integral in $uv-$coordinates; use as the order of integration $dv\ du$.