18.02A Problem Set 6 - Spring 2007

Due Friday, Feb. 23 at 2:00

Part I (15 points)

Lecture 45. (*Fri., Feb. 16*) Exact differentials; Potential functions.
Read: Notes V2 *Work:* 4C-5b, 6b
Lecture 46. (*Tues., Feb. 20*) Recitation only due to President's Day Holiday; Meet in room 2-142 at 2:00.
Lecture 47. (*Thurs., Feb. 22*) Green's Theorem (tangential form).
Read: Simmons 21.3 *Work:* 4D-1bc, 3, 5
Lecture 48. (*Frid., Feb. 23*) **Exam** covering lectures 40–45; in class.

Part II (10 points)

Try each problem alone for 15 minutes before collaborating, and write up solutions independently. The problems are given in order according to the lecture schedule above.

Problem 1. (*2 pts: 1+1*) Consider the vector field

$$\mathbf{F} = (2xyz + 2y^2\cos 2x)\,\hat{\mathbf{i}} + (x^2z + 2y\sin 2x)\,\hat{\mathbf{j}} + (x^2y + e^z)\,\hat{\mathbf{k}}.$$

a) Find a potential function $f(x, y, z)$ using the algebraic method.
b) Find a potential function $f(x, y, z)$ using the integration method.

Problem 2. (*4 pts: 1+1+2*) For any values of a and b, define

$$\mathbf{F} = x^a y^b\,\hat{\mathbf{i}} + x^b y^a\,\hat{\mathbf{j}}.$$

a) Calculate the curl of $\mathbf{F}$.
b) Determine what values of a and b make $\mathbf{F}$ a gradient field.
c) Find potential functions $f(x, y)$ for all allowable pairs of a and b from part b). *Hint: You may need to specify the domain of* $\mathbf{F}$ *for negative* a *and* b; *one pair requires especially careful attention.*

Problem 3. (*2 pts: 1+1*) Let $\mathbf{c}$ be the path from $(-1, -1)$ to $(0, 0)$ that follows the curve $y = x^3$, and suppose that $\mathbf{F} = \vec{\nabla}(x^4 + 2x^3y - \frac{y^4}{2})$.

a) Evaluate $\int_{\mathbf{c}} \mathbf{F} \cdot dr$ by using the Fundamental Theorem of Calculus for line integrals.
b) Evaluate $\int_{\mathbf{c}} \mathbf{F} \cdot dr$ by appealing to path-independence to replace $\mathbf{c}$ by an alternate path. *Hint: Try a straight line.*

Problem 4. (*2 pts*) If the path $\mathbf{c}$ follows the positively oriented triangle with vertices $(-2, 0), (0, 1)$, and $(2, 0)$, use Green's Theorem to evaluate

$$\oint_{\mathbf{c}} (x^3y^2 - x\cos y)\,dx + \left(\frac{x^4y}{2} + \frac{x^2}{2}\sin y - xy\right)\,dy.$$