# 18.02A Problem Set 7 - Spring 2007

Due Friday, Mar. 2 at 2:00

**Part I** (15 points)

**Lecture 49.** (*Tues., Feb. 27*) Flux; Normal form of Green's Theorem; Applications.
*Read:* Notes V3, V4 *Work:* 4E-1ab, 3; 4F-3, 5
**Lecture 50.** (*Thurs., Mar. 1*) Surface integrals; Surface area.
*Read:* Notes V9 (ignore flux) *Work:* 6B-2, 3, 4, 12
**Lecture 51.** (*Fri., Mar. 2*) Divergence Theorem.
*Read:* Notes V10

**Part II** (10 points)

Try each problem alone for 15 minutes before collaborating, and write up solutions independently. The problems are given in order according to the lecture schedule above.

**Problem 1.** (*3 pts: 2+1* ) In this problem, all curves $\mathbf{c}$ are restricted to lie only in the first quadrant $(x, y \geq 0)$.
a) Find the positively oriented curve that maximizes

$$\int_{\mathbf{c}} \left( xy + \frac{y^2}{2} \right) \, dx + (2x - ye^y) \, dy.$$

(*Hint: Use Green's Theorem in tangential form to replace the line integral with a double integral*)
b) Calculate the maximum value of the integral along the path found in part a).

**Problem 2.** (*2 pts: 1+1* ) Consider the flow field

$$\mathbf{F} = (-x + 2y \sin x - \sqrt{1 + y^3}) \, \hat{\mathbf{\imath}} + (e^x + y^2 \cos x + 3y) \, \hat{\mathbf{\jmath}}.$$

a) Calculate the divergence of $\mathbf{F}$.
b) Let $\mathbf{c}$ be the clockwise path around the triangle with vertices $(0, 0)$, $(0, b)$, and $(a, b)$. Use Green's Theorem to calculate the total flux across $\mathbf{c}$.

**Problem 3.** (*2 pts*) Calculate the flux of $\mathbf{F} = (4xy^3 + 2xy + 3y^2) \, \hat{\mathbf{\imath}} + (2 - y^4) \, \hat{\mathbf{\jmath}}$ around the semicircle of radius 1 that travels in the clockwise direction from $(0, 1)$ to $(0, -1)$. (*Hint: Use Green's Theorem in normal form; be aware that the given curve is not closed.*)

**Problem 4.** (*3 pts: 2+1* ) a) A *torus* is the "doughnut" shape that is obtained as the surface of revolution of a disc. In particular, suppose that the center of a disc of radius 1 is placed a distance $R$ from the $z$-axis, and is then rotated about this axis. Write down a parameterization for this surface.
(*Hint: Start with a circle in the* $(y, z)$ *plane parameterized by* $u$ :

$$y = R + \cos u, \qquad z = \sin u, \qquad 0 \leq u \leq 2\pi.$$

*Now use a new parameter* $v$ *to rotate around the* $z$*-axis.*)

b) Write (but do not evaluate!) the surface integral that calculates the surface area of the torus. (*Note: The integral evaluates to* $4\pi^2 R$.)