

Problem Solving Seminar - Fall 2012
Oct. 15

1. (a) Find the area of the region

$$\{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq y \leq 1\}.$$

- (b) Find the volume of the region

$$\{(x, y, z) \in \mathbb{R}^3 \mid 0 \leq x \leq y \leq z \leq 1\}.$$

2. Calculate the volume of a tetrahedron $ABCD$ with side-lengths $AB = AC = AD = 5$ and $BC = 3$, $CD = 4$, and $BD = 5$.
3. (a) Find a way to cut a 9×16 rectangle into two pieces that can be assembled into a 12×12 square.
(b) Does your construction generalize? Can you cut a 16×25 rectangle into two pieces that form a 20×20 square?
4. A coin of diameter d is dropped on a tile floor that consists of $d \times d$ squares. What is the probability that the coin covers one of the corners on the floor?
5. [1998 A-1] A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?
6. [1998 A-2] Let s be any arc of the unit circle lying entirely in the first quadrant. Let A be the area of the region lying below s and above the x -axis and let B be the area of the region lying to the right of the y -axis and to the left of s . Prove that $A + B$ depends only on the arc length, and not on the position, of s .