

**Problem Solving Seminar - Fall 2012**  
**Oct. 22**

1. Suppose that 5 dice are rolled.
  - (a) What is the probability that their sum is 4?
  - (b) What is the probability that their sum is at most 6?
2. If  $a$  and  $c$  are random real numbers between  $-2$  and  $2$ , find the probability that the parabola  $ax^2 - 2x + c$  is positive for some  $x$ .
3.
  - (a) A picnic table seats 4 people on each of its two sides. If 2 men and 4 women sit at random positions around the table, what is the probability that the men are on opposite sides?
  - (b) Now suppose that 2 men and 8 women sit at a square table that seats 4 people on each side. What is the probability that the men are on opposite sides?
  - (c) What if the table is rectangular, with  $m$  seats the long way, and  $n$  seats the short way?
4. The cards of a standard 52-card deck are flipped over one at a time.
  - (a) What is the probability that the Ace of Spades is the first card?
  - (b) What is the probability that the Ace of Spades is the 32-nd card?
  - (c) What is the probability that the last card is a Heart?
  - (d) What is the probability that the Ace of Spades appears before any Heart?
5. **[2001 A-2]** You have coins  $C_1, C_2, \dots, C_n$ . For each  $k$ ,  $C_k$  is biased so that, when tossed, it has probability  $1/(2k + 1)$  of falling heads. If the  $n$  coins are tossed, what is the probability that the number of heads is odd? Express the answers as a rational function of  $n$ .
6. **[1993 B-3]** Two real numbers  $x$  and  $y$  are chosen at random in the interval  $(0, 1)$  with respect to the uniform distribution. What is the probability that the closest integer to  $x/y$  is even? Express the answer in the form  $r + s\pi$ , where  $r$  and  $s$  are rational numbers.