

Problem Solving Seminar - Fall 2012
Nov. 12

1. (a) Characterize the integers n for which $n^3 + 1000$ is divisible by $n + 10$.
(b) Characterize the integers n for which $n^2 + 100$ is divisible by $n + 10$.
2. Suppose that $x + y = 1$ and $xy = -1$.
 - (a) What is the value of $x^2 + y^2$?
 - (b) What is the value of $x^3 + y^3$?
 - (c) What is the value of $x^{10} + y^{10}$?
Hint: Try to find a recurrence relation for $L_n := x^n + y^n$.
3. On a 3×7 board, every square is colored Red or Green. Show that in any such coloring, there must be rectangle (formed by the lines of the board) whose distinct corner squares are all the same color).
4. Let $p(x)$ be a real polynomial such that $p(x) + p'(x) \geq 0$ for all x . Does it follow that $p(x) \geq 0$ for all x ?
5. [1971 A-2] Determine all polynomials $P(x)$ such that $P(x^2 + 1) = (P(x))^2 + 1$ and $P(0) = 0$.
6. [1997 B-2] Let $f(x)$ be a twice-differentiable real-valued function satisfying

$$f(x) + f''(x) = -xg(x)f'(x),$$

where $g(x) \geq 0$ for all real x . Prove that $|f(x)|$ is bounded.