

Problem Solving Seminar - Fall 2012
Nov. 19

1. Suppose that n is a positive integer.
 - (a) Are there any values of n such that $n^4 + 2n^3 + 2n^2 + 2n + 1$ is a perfect square?
 - (b) Suppose that every digit of n is 0 or 6. Is it possible that n is a perfect square? A perfect cube? A perfect fourth power?
2.
 - (a) What is the largest value of the function $f(\theta) = \cos(\theta) + \sin(\theta)$?
 - (b) What is the largest value of the function $f(\theta) = \cos(\theta) + 2\sin(\theta)$?
3.
 - (a) Factor $z^5 + z + 1$ into two polynomials with integer coefficients.
 - (b) Find all roots of $z^6 + z^4 + z^3 + z^2 + 1 = 0$.
Hint: What happens if z is a complex 5-th root of unity?
4. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find all values λ and vectors \mathbf{x} such that $A\mathbf{x} = \lambda\mathbf{x}$.
5. Suppose that A and B are $n \times n$ matrices.
 - (a) If $A^2 = A$ and A is nonzero, must A be invertible?
 - (b) Prove that if $AB = 0$ and B is nonzero, then A is not invertible.
6. [1991 A-2] Suppose that A and B are two different $n \times n$ matrices with real entries that satisfy $A^3 = B^3$ and $A^2B = B^2A$. Is it possible that $A^2 + B^2$ is invertible?
7. [1990 A-5] If A and B are square matrices of the same size such that $ABAB = 0$, does it follow that $BABA = 0$?