1. At LSU, 91% of students are fans of Tiger Football, 68% are fans of Baseball, and 44% are fans of Basketball*. Prove that there is at least one "Super-Fan" who supports all three sports.

   * Statistics may not be accurate.

2. Suppose that N dice are thrown.

   (a) What is the probability that their product is even?

   (b) What is the probability that their sum is even?

   (c) What is the probability that their product and sum are both odd?
       Hint: This is very easy to answer if N is even!

   (d) What is the probability that their product and sum are both even?

3. A kitchen floor is covered with square tiles that measure 9 inches per side. If a large pizza (diameter 18 inches) is randomly dropped on the floor, what is the probability that it covers exactly two corners of some tile?

4. The cards of a standard 52-card deck are turned over one at a time.

   (a) What is the probability that the first card is a Heart?

   (b) What is the probability that the last card is a Spade?

   (c) What is the probability that the first card is a Heart and the last card is a Spade?

   (d) What is the probability that the first card is a Heart and the last card is the Ace of Spades?

5. In many professional sports leagues (Baseball, Basketball, Hockey) the championship is determined by a "Best of 7" series, in which the first team to win 4 games is the victor. If the two teams are evenly matched, calculate the probability that a series ends in exactly $4, 5, 6,$ or 7 games.

6. **[2006 A4]** Let $S = \{1, 2, \ldots, n\}$ for some integer $n > 1$. Say a permutation π of S has a local maximum at $k \in S$ if

   (i) $\pi(k) > \pi(k+1)$ for $k = 1$;
   (ii) $\pi(k-1) < \pi(k)$ and $\pi(k) > \pi(k+1)$ for $1 < k < n$;
   (iii) $\pi(k-1) < \pi(k)$ for $k = n$.

   (For example, if $n = 5$ and π takes values at $1, 2, 3, 4, 5$ of $2, 1, 4, 5, 3$, then π has a local maximum of 2 at $k = 1$, and a local maximum of 5 at $k = 4$.) What is the average number of local maxima of a permutation of S, averaging over all permutations of S?

Challenge.

1. Suppose that N dice are thrown.

   (a) What is the probability that their sum is a multiple of 6?

   (b) What is the probability that their sum is a multiple of 5?

   (c) What is the probability that their sum is a multiple of 7?