MATH 7290 Homework 2 - Spring 2013

Due Thursday, Feb. 14 at 1:30

The notation "Koblitz A.B.C" means Problem C from Chapter A, Section B of the textbook.

[Tues., Jan. 29]

1. Prove that the Mordell-Weil addition law is associative. In particular, suppose that $P_i = (x_i, y_i)$ for $i = 1, 2, 3$ are rational points on $E: \ y^2 = x^3 + ax + b$, and show that
$$(P_1 + P_2) + P_3 = P_1 + (P_2 + P_3)\,.$$
2. Koblitz 1.7.2.

[Thurs., Jan. 31]

3. Koblitz 1.7.4. Note that part (a) refers to the *complex* inflection points on an elliptic curve.
4. Find a birational equivalence between the quartic curve $u^4 + u^2v^2 + v^4 = 0$ and the elliptic curve $y^2 = x^3 + 4x$.

[Tues., Feb. 5]

5. Euler gave the following product expansion for the sine function:
$$\sin(x) = x \prod_{n=1}^{\infty} \left(1 - \frac{x^2}{\pi^2 n^2}\right),$$
with the "proof" that since $\sin(x)$ has zeroes at all integer multiples of π, there must be some constant C such that $\sin(x) = C \prod_{n\in\mathbb{Z}} (x - \pi n)$.
   (a) What is wrong with Euler's proof?
   (b) Regardless, the formula is correct (though quite an exercise in analysis to prove!). Calculate the logarithmic derivative
$$\frac{d}{dx} \log\left(\sin(x)\right)$$
and compare with the Laurent series for the cotangent from lecture.

[Thurs., Feb. 7]

6. Koblitz 1.5.1. Recall from lecture that the question is only in regards to the points of the lattice; the fundamental parallelogram associated to $L(\omega_1, \omega_2)$ and $L\left(\omega_1', \omega_2'\right)$ will be different.

7. Koblitz 1.5.2.

8. Complete the proof that the Weierstrass function $\mathcal{P}_L(z)$ is L-periodic by verifying the following:

   (a) $\mathcal{P}_L(z+\omega_i) = \mathcal{P}_L(z) + \sum_{\substack{\ell \in L \\ \ell \neq 0, -\omega_i}} \frac{1}{(\ell+\omega_i)^2} - \frac{1}{\ell^2}$.

   (b) The sum $\sum_{\substack{\ell \in L \\ \ell \neq 0, -\omega_i}} \frac{1}{(\ell+\omega_i)^2} - \frac{1}{\ell^2}$ equals 0.