Upcoming dates:

- Virginia Tech Regional Mathematics Contest. Saturday, Oct. 25, 9:00 – 11:30 A.M., in Lockett Hall. **Sign-up deadline: Oct. 1**.
- William Lowell Putnam Mathematical Competition. Saturday, Dec. 6, 9:00 A.M. – 5:00 P.M., in Lockett Hall. **Sign-up deadline: Oct. 8**.

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Problem Solving Seminar - Fall 2014
Sep. 17

1. Prove that every difference in the following list is a perfect square:

$$\begin{gathered} 11-2, \\ 1111-22, \\ 111111-222, \\ \vdots \end{gathered}$$

2. Denote an integer expressed in base b by $(d_k d_{k-1} \cdots d_1 d_0)_b$. For example, in base 3 we have

$$221_3 = 2 \cdot 3^2 + 2 \cdot 3^1 + 1 \cdot 3^0 = 25.$$

   (a) Determine which is larger:

$$12345_8 \qquad \text{or} \qquad 543216_6?$$

   (b) Evaluate the product

$$22 \cdots 2_5 \times 22 \cdots 3_5,$$

   where the two numbers contain $(n-1)$ 2s followed by a 2 and a 3.

   (c) Evaluate the product

$$44 \cdots 4_9 \times 44 \cdots 5_9,$$

   where the two numbers contain $(n-1)$ 4s followed by a 4 and a 5.

   (d) Factor the integers

$$2_9, \quad 222_9, \quad 22222_9, \ldots,$$

   where each number contains an *odd* number of 2s.

3. A popular style of arithmetic puzzle is a *Cryparithm*, where each letter is replaced by a digit so that the resulting expression is true. Solve the following:

(a)

$$\begin{array}{rccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline & M & O & N & E & Y \end{array}$$

(b)

$$\begin{array}{rccccc} & F & O & R & T & Y \\ & & & T & E & N \\ + & & & T & E & N \\ \hline & S & I & X & T & Y \end{array}$$

4. A woman has an extraordinary Social Security number, which contains each of the nine digits 1 to 9 exactly once. Furthermore, the first two digits (read left-to-right) are a multiple of 2, the first three digits are a multiple of 3, and so on, until the complete number is a multiple of 9.

   How many possible Social Security numbers have these properties?

5. Let N be the positive integer with 2014 decimal digits, all of them 1; that is,

$$N = 111 \cdots 11.$$

   Let M be the largest integer that is smaller than or equal to $\sqrt{N}$ Find the sum of the digits of M.

6. [Putnam **1998 B5**] Let N be the positive integer with 1998 decimal digits, all of them 1. Find the thousandth digit after the decimal point of $\sqrt{N}$.

Challenge.

1. (a) Prove that there are no perfect squares among the integers that contain a single digit repeated multiple times; in particular, these are the numbers

$$\begin{array}{cccc} 11, & 111, & 1111, & \ldots, \\ 22, & 222, & 2222, & \ldots, \\ & \vdots & & \\ 99, & 999, & 9999, & \ldots. \end{array}$$

   (b) If the numbers are written in base b instead of base 10, then there are perfect squares. For example, there is a base b such that 11_b and 11111_b are square - find such a b? Describe the general behavior as thoroughly as you can.