

## Problem Solving Seminar - Fall 2014

Oct. 22

1. Suppose that 3 dice are rolled.
  - (a) What is the probability that the sum is even?
  - (b) What is the probability that the sum is at least 11?
  - (c) Would the answer to part (a) be different if 4 dice were rolled?

*Hint: Or 2 dice?*
2. At LSU, 52% of students Love Purple, 73% Live Gold, and 88% say "Geaux Tigers!"\*. Prove that there is at least one student who supports all three mottos.

\* Statistics may not be accurate.

3. The cards of a standard 52-card deck are turned over one at a time.
  - (a) What is the probability that the first card is a Heart?
  - (b) What is the probability that the last card is the Queen of Hearts?
  - (c) What is the probability that a Heart is turned over before any Spade?
4. A stick of length one is cut at two random points (all points being equally likely). What is the probability that the resulting three pieces can be arranged to form a triangle?
5. [Gelca-Andreescu **910**] A bag contains 1993 Red balls and 1993 Black balls. We remove two balls at a time repeatedly and
  - (a) Discard them if they are the same color,
  - (b) Discard the Black ball and return the Red ball to the bag if they are different colors.

What is the probability that this process ends with an empty bag?

6. (a) Three points are randomly chosen on a circle, and are connected to form a triangle. What is the probability that the center of the circle is contained in the interior of the triangle?

*Hint: Without loss of generality, the circle can be rotated so that the first point is fixed. You therefore only need choose **two** points randomly.*
- (b) [Gelca-Andreescu **931**] What is the probability that three randomly chosen points (on a circle) are all contained in some semicircle?

7. [VTRMC **2004 #3**] A computer is programmed to randomly generate a string of six symbols using only the letters  $A, B, C$ . What is the probability that the string will not contain three consecutive  $As$ ?
8. [Putnam **1989 B1**] A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge. Express your answer in the form  $\frac{a\sqrt{b} + c}{d}$ , where  $a, b, c, d$  are integers.

### Challenge.

1. Recall Problem 6.

- (a) Suppose that 4 points are randomly chosen on a sphere, and are connected to form a tetrahedron. What is the probability that the center of the sphere is in the interior of the tetrahedron?
- (b) Answer the same question for  $n + 2$  points randomly chosen on an  $n$ -dimensional hypersphere.