LSU Problem Solving Seminar - Fall 2015
Sep. 2: Classic Puzzles

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<u>Warm Up:</u>

1. (a) A villager plans to prepare a large pasta dinner, and in order to cook it perfectly he will need four gallons of water. However, his only two buckets hold three and five gallons of water. How can he return from the river with exactly four gallons of water?

   (b) What if he instead had two buckets that hold five and six gallons of water – is it still possible to measure exactly four gallons?

2. It requires one minute per side to cook a piece of Texas toast in a pan. If the pan can only hold two pieces of bread at a time, how quickly can 3 pieces of toast be prepared?

3. A frog lives at the bottom of a well that is 20 feet deep. One day she wishes to escape and see the world. She climbs 3 feet up the wall each day, but as she sleeps she slides 2 feet back down the slimy walls each night. On which day does she emerge at the top?

4. Consider the following three statements:

   (a) There is one false statement in this list.

   (b) There are two false statements in this list.

   (c) There are three false statements in this list.

   If this collection is logically consistent, which of the statements is true?

<u>Main Problems:</u>

5. Suppose that your sock drawer contains 12 White socks, 8 Gold Socks, and 15 Purple Socks. You are leaving very early in the morning, and cannot turn on the light to look at the colors without waking up your roommate.

   (a) How many socks must you grab in order to guarantee that you have a matching pair?

   (b) How many socks do you need to guarantee **two** matching pairs?

6. (**"The Ant and the Plant"**). An ant approaches the base of a plant that is 1 meter tall and begins climbing, ascending 1 centimeter during the course of the day. That night the plant grows 1 centimeter, with the growth distributed proportionally. In other words, since the height of the plant is now 1.01 meters, the ant wakes up at a height of 1.01 cm. The ant again climbs a total of 1 cm the next day, and the plant grows an additional 1 cm the second night, stretching proportionally as the ant sleeps.

   If this process continues, will the ant ever reach the top of the plant?

7. Alice, Bob, Charlie, and Diane reach a rickety bridge in the middle of the night that is too dangerous to cross in the dark. The bridge is narrow and can only accommodate two people crossing at once. Furthermore, the group only has one torch, which will burn for just 20 minutes. Each person has a different maximum crossing speed: 2 minutes for Alice, 3 for Bob, 7 for Charlie, and 8 for Diane. Note that when two people are crossing together, they must travel at the slower person's speed so that they can share the torch.

   Is it possible for all four people to cross within the 20 minutes of available torchlight?

8. You are sitting in a dark room. You are handed a standard deck of 52 cards, and are told that the Hearts are face-up, with all other suits face-down. Finally, you are asked to arrange the deck into two stacks such that each one contains an equal number of cards that are face-up.

   Hint: Consider a simpler case: What if you were given a deck with just one card face-up?

9. [Putnam **1965 A5**] In how many ways can the integers from 1 to n be ordered subject to the condition that, except for the first integer on the left, every integer differs by ± 1 from some integer to the left of it?