LSU Problem Solving Seminar - Fall 2016
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Warm Up

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1. (a) You enter a classroom where a partially erased equation was left on the board:

$$1 \_\_ 2 \_\_ 3 \ = 6.$$

Note that this equation is true if both missing symbols are replaced by "+" or by "×". Are there any other integers a, b, c and d that also have this property: namely, that

$$a \_\_ b \_\_ c \ = d$$

holds for both addition and multiplication?

(b) On the second day you enter a classroom where a partially erased equation was left on the board:

$$\big(6 \_\_ 2\big) \_\_ \big(4 \_\_ 3\big) \_\_ \big(6 \_\_ 2\big) = 25.$$

Fill in the blanks with arithmetic symbols so that the resulting equation is true.

(c) On the final day the same equation is still on the board, with your solution erased, and an added message:

Nice job, but there are at least **two** solutions...

2. If one carpenter can craft a door in 10 hours, how long will it take 4 carpenters to complete 6 doors?

3. (a) An Olympic cyclist trains one day by riding for one hour at 20 miles per hour, and then a second hour at 30 miles per hour. How many total miles did he ride?

(b) The next day he rides a 25-mile course at 20 miles per hour, and then turns around and returns at 30 miles per hour. How long does it take him in total?

Main Problems

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4. Find all n such that $\left(\frac{11}{2}\right)^n + \left(\frac{5}{2}\right)^n$ is an integer.

5. (a) [Gelca-Andreescu **5**] The union of nine planar surfaces, each of area equal to 1, has a total area equal to 5. Prove that the overlap of some two of these surfaces has an area greater than or equal to $\frac{1}{9}$.

(b) Draw a planar region with area 2 and color it with Red, Yellow, and Blue (with overlap) in such a way that

- Each color covers an area of 1; and

- Each two colors occur together in an area of at most $\frac{1}{3}$.

(c) Generalize the problem as much as you can... what if there are n surfaces?

6. Given a finite collection of squares with total area at least 3, show that they can be arranged without rotation (so that all sides remain parallel) to cover a square of side length 1.

   Note: This result is sharp – for example, with three squares of side length 0.99*, one of the four corners of the unit square cannot be covered!*

7. [Putnam **2010 B2**] Given that A, B, and C are noncollinear points in the plane with integer coordinates such that the distances AB, AC and BC, are integers, what is the smallest possible value of AB?