- Virginia Tech Mathematics Contest. Sat., Oct. 22. **Sign-up deadline: Sep. 30**.
- Putnam Mathematical Competition. Sat., Dec. 3. **Sign-up deadline: Oct. 7**.

LSU Problem Solving Seminar - Fall 2016
Aug. 31

Prof. Karl Mahlburg
Website: www.math.lsu.edu/∼mahlburg/teaching/2016-Putnam.html

Warm Up

1. One day a single cell of bacterial film grows on the surface of a pond. Each day the bacteria population doubles, so that on the second day there are 2 cells, on the third day 4 cells, and so on. The pond is completely covered in bacteria on the 30-th day. On which day was exactly half of the pond covered by bacteria?

2. (a) Is there a triangle with side lengths $(8, 5, 7)$? Is there a triangle with side lengths $(2, 11, 10)$? How about $(5, 7, 12)$?
   (b) Suppose that a, b, and c are positive real numbers. Give a simple condition that determines whether or not there is a triangle with side lengths (a, b, c).
   (c) Find a condition that determines whether there is a quadrilateral with side lengths (a, b, c, d). Is there a quadrilateral with side lengths $(3, 10, 4, 12)$?

3. At a remote island hospital a patient requires three emergency surgical procedures, to be performed by Doctors Adams, Brooks, and Charles. Unfortunately, this island is home to an extremely contagious disease that can be spread by any contact of skin or body fluids. Even worse, supplies are low, and the hospital only has two pairs of sterile surgical gloves, but the surgeries must be performed immediately.

   The doctors find a clever plan to safely perform the surgeries: Dr. Adams will perform his surgery wearing the second pair of gloves on top of the first. Afterwards he removes them, turning the first pair inside out. Now Dr. Brooks can wear the second pair of gloves for her surgery, and Dr. Charles can wear the inverted first pair of gloves.

   However, the nurse is horrified when he hears this, as he points out that if Dr. Adams is infected, the patient will be exposed! Find an alternative way to use the two pairs of gloves that prevents any possible contamination amongst the doctors and the patient.

Main Problems

4. Insert the arithmetic symbols $+, -, \times$, and any necessary parentheses so that the following expression is correct

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 = 2016.$$

Hint: See bottom of reverse page for part of one possible solution.

5. [Gelca-Andreescu **58**] Complete the square with integers between 1 and 9 such that the sum along each row, column, and diagonal gives the circled total.

		2		(14)
	5			(16)
			8	(26)
3				(30)
(16)	(21)	(25)	(13)	(27)	(20)

6. Suppose that a clock's minute hand is precisely on a minute mark, and is exactly 12 minutes ahead of the hour hand. What time is it?

7. 16 numbered checkers are placed on a 4×4 grid as below. Your goal is to perform a sequence of moves so that the checkers are in order from left-to-right: $1, 2, 3, 4$ on the first row, $5, 6, 7, 8$ on the second row, and so on. Each move consists of swapping two checkers. Determine the **minimum** number of moves necessary to achieve the desired order.

(16)	(11)	(2)	(5)
(13)	(8)	(4)	(9)
(12)	(6)	(3)	(10)
(1)	(15)	(14)	(7)

Generalize your answer: Write down another 4×4 array and find the minimum number of moves.

8. [Putnam **1996 B5**] Given a finite string S of symbols X and O, write $\Delta(S)$ for the number of Xs minus the number of Os. For example, $\Delta(XOOXOOX) = -1$. A string S is **balanced** if every substring T of (consecutive symbols of) S has $-2 \leq \Delta(T) \leq 2$. Thus, $XOOXOOX$ is not balanced since it contains the substring $OOXOO$. Find, with proof, the number of balanced strings of length n.

Hint for Problem 4: 1 2 3 $(4-5)+6$ 7 8×9.