- Virginia Tech Mathematics Contest. Sat., Oct. 21. **Sign-up deadline: Sep. 30**.
- Putnam Mathematical Competition. Sat., Dec. 2. **Sign-up deadline: Oct. 6**.

LSU Problem Solving Seminar - Fall 2017
Sep. 6: Classic Puzzles

Prof. Karl Mahlburg
Website: `www.math.lsu.edu/~mahlburg/teaching/Putnam.html`

Warm Up

1. Alice is currently 2 years younger than 3 times her younger brother Bob's age. In 2 years, she will be exactly twice as old as Bob. Last year she was 3 times older than her brother. What are their current ages?

2. (a) Consider the following list of statements:
   (i) There is 1 true statement in this list.
   (ii) There are 2 true statements in this list.
   (iii) There are 3 true statements in this list.
   If the collection of statements is logically consistent, which one(s) is/are true?

   (b) Consider the following list of statements:
   (i) There is 1 true statement in this list.
   (ii) There are 2 true statements in this list.
   (iii) There is 1 false statement in this list.
   (iv) There are 2 false statements in this list.
   (v) There are 3 false statements in this list.
   If the collection of statements is logically consistent, which one(s) is/are true?
   Remark: There are two possible answers in this case.

3. 100 ants are placed on a narrow stick that measures 1 meter long. Each ant begins marching in a random direction (left or right), and maintains a constant speed of 1 cm per second. Whenever two ants bump into each other from opposite directions, they both immediately turn around and continue walking. What is the maximum amount of time it will take for all of the ants to fall off the ends of the stick?

Main Problems

4. (**Magic Squares**.)

   (a) Fill in the grid below with the numbers $1-9$ to construct a *Magic Square*: this means that the sum along each row and each column is identical.

Try to be systematic in your construction – no guessing!

Hint: If you place 1 *in the center, what can you conclude about its neighbors?*

(b) There are $9! = 362880$ ways to place the digits $1-9$ in the grid. How many of them are Magic Squares?

(c) Can you find a *Super Magic Square*, where the sums along the diagonals also match the row and column sums?

5. Three large boxes of LSU t-shirts are delivered, and are labeled Purple, Gold, and Purple/Gold (an even mix of each color). However, there is also a note from the factory explaining that although there is one box each of Purple, Gold, and Purple/Gold, each box is incorrectly marked due to a malfunction with the labeler. How can you determine the correct labels by opening **one** box and taking out **one** shirt?

6. (**Three Hats Problem.**) Three mathematicians are led into a darkened room where a total of 5 hats sit on a table: 3 **Black** hats, and 2 **White** hats, and each puts on a hat. When they exit back into a lit room (with no mirrors!), the first looks at the other two, and says "I do not know what color my hat is." The second soon replies "I do not know the color of my hat either." Finally, after hearing both of these comments, the third person thinks a bit longer, and says "Now I **do** know the color of my hat."

   (a) What color is the third person's hat?

   (b) If the second person then says "I still do not know the color of my hat," what color is the first person's hat?

7. (a) Using a precisely calibrated compass, you walk one mile due South, then one mile due West, and finally one mile due North. Surprisingly, you ended up at exactly the same point you started! Where did you start?

   (b) Now find another solution!

   Hint: The "obvious" solution is not on solid ground depending on the time of year, but there are other solutions where it is always possible to walk.

8. (**Josephus Problem.**) Among a group of n prisoners condemned to death, one will be set free according to the following procedure. The prisoners are arranged in a circle and numbered $1, 2, \ldots, n$, and beginning from person 1, the warden goes around the circle and removes every second person. For example, if there are 6 people, then the first time around, $2, 4$ and 6 are removed, leaving $1, 3, 5$. The second remaining person is now 3, who is removed, and the second after that is 1, leaving 5 to be set free.

   (a) Which prisoner is set free if $n = 2^k$?

   (b) Which prisoner is set free if $n = 2^k + 1$?

9. [Putnam **1997 A2**] Players $1, 2, 3, \ldots, n$ are seated around a table, and each has a single penny. Player 1 passes a penny to player 2, who then passes two pennies to player 3. Player 3 then passes one penny to Player 4, who passes two pennies to Player 5, and so on, players alternately passing one penny or two to the next player who still has some pennies. A player who runs out of pennies drops out of the game and leaves the table. Find an infinite set of numbers n for which some player ends up with all n pennies.