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- Virginia Tech Mathematics Contest. Sat., Oct. 27. **Sign-up deadline: Sep. 28.**
 - Putnam Mathematical Competition. Sat., Dec. 1. **Sign-up deadline: Oct. 5.**
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LSU Problem Solving Seminar - Fall 2017
Aug. 22

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Warm Up

1. Last year John was four times as old as his sister Alice. In three years he will be twice as old as her. How old are John and Alice currently?
2. (a) You enter a classroom where a partially erased equation was left on the board:

$$2 _ 2 = 4.$$

Note that this equation is true if the missing symbols is “+” or by “×”. Are there any other integers a, b , and c that also have this property: namely, that

$$a _ b = c$$

holds for both addition and multiplication?

- (b) On the second day you enter a classroom where another partially erased equation was left on the board:

$$a _ b = a _ b.$$

Note that this is also true with $a = b = 2$ if the missing operations are \times and \wedge ; i.e., $2 \cdot 2 = 2^2$. Are there any other integers a, b such that $ab = a^b$?

- (c) On the final day the following equation is on the board:

$$a + b = a \times b = a \wedge b.$$

Find all integer solutions.

3. Suppose that everyone in the room will be randomly given a Purple or Gold hat. Assume that there are no mirrors or reflective materials, so that you will not be able to see the color of your own hat, although you can see the color of everyone else’s hats. Your collective goal is for one person to move at a time such that all Purple hats are on one side of the room, and all Gold hats are on the other side. You are not allowed to communicate once the hats are distributed, so you must all decide on a strategy in advance.

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4. Let $f(x) := e^{x^2}$.
- (a) Calculate $f''(x)$.
 - (b) Calculate $f'''(x)$.
 - (c) Evaluate $f^{(5)}(0)$. Try to do this **without** explicitly writing down the general formula for the fifth derivative!
5. [From **Gelca-Andreescu (1st) 29**] Suppose that $P = P_1 \cdots P_n$ is a polygon (not necessarily convex). Is it always possible to dissect P into triangles by interior diagonals?
6. Is it true that $e^x \geq ex$ for all real x ?
- Hint: This is equivalent to $x \geq 1 + \ln(x)$.*
7. [VTRMC **1980 # 5**] For $x > 0$, show that $e^x < (1+x)^{1+x}$.
- Hint: Start by considering the range $x \geq e - 1$.*
8. [Putnam **2004 A2**] For $i = 1, 2$, let T_i be a triangle with side lengths a_i, b_i, c_i , and area A_i . Suppose that $a_1 \leq a_2, b_1 \leq b_2, c_1 \leq c_2$, and that T_2 is an acute triangle. Does it follow that $A_1 \leq A_2$?
9. Note that $2^5 = 32$. Are there any other powers of 2 whose decimal expansions consists of a sequence of 3's followed by a sequence of 2's?
10. [Putnam **1956 A2**] Prove that every positive integer has a multiple whose decimal representation involves all ten digits 0 – 9.