- Virginia Tech Mathematics Contest. Sat., Oct. 27. **Sign-up deadline: Sep. 28**.
- Putnam Mathematical Competition. Sat., Dec. 1. **Sign-up deadline: Oct. 5**.

LSU Problem Solving Seminar - Fall 2018
Aug. 29

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Website: www.math.lsu.edu/∼mahlburg/teaching/Putnam.html

Warm Up

1. (a) If 10 painters can finish 1 floor of a building in 10 minutes, how long will it take 50 painters to finish 5 floors?
   (b) If an experienced painter can finish a unit of wall in 15 minutes, while a new painter requires 30 minutes, how long will it take them to finish a unit of wall when they work together?

2. A visitor to the Island of Uncertainty meets a Priest and a Monk in the forest. The visitor knows that Priests always lie on Monday, Tuesday, and Wednesday, and tell the truth on other days, while Monks always lie on Thursdays, Fridays, and Saturdays, and are truthful on other days. Unfortunately, the visitor has been lost for weeks, and no longer knows what day it is. They speak to her as follows:

   Priest: Yesterday I was lying.
   Monk: Yesterday I was also lying.

   On which day of the week did she meet them?

3. Three Merchants and three Thieves come to a river crossing where the available boat can only hold two people. The Thieves will cooperate with the Merchants with one exception: if there are ever more Thieves than Merchants on one side of the river, the Merchant(s) will be beaten and robbed. How can the Merchants cross the river safely?

Main Problems

4. (**Beetle Problem.**) Nine beetles are placed on a circular track such that the distances between consecutive beetles are the first nine prime numbers: $2, 3, 5, 7, 11, 13, 17, 19, 23$. After the beetles are all in position, they each begin moving in a random direction. Once a beetle is walking in a certain direction, it will continue in that direction until it runs into another beetle, at which time they will both change directions. If the beetles move at 1 unit of distance per minute, prove that after 50 minutes there will be two adjacent beetles that are 13 units apart.

   Hint: In fact, **much** *more can be said about the final configuration. . . .*

5. (**Elephant and Bananas**.) There is a severe shortage of Bananas in Texas, and Louisiana has released 4500 pounds from its Strategic Banana Reserve. Unfortunately, the only available mode of transportation is an Elephant who can carry a maximum of 1500 pounds of cargo. Furthermore, he must eat 10 pounds of Bananas for each mile traveled. Since it is 150 miles from Baton Rouge to the Texas border, it will not work to simply send the Elephant with 1500 pounds of Bananas all the way to Texas – all of Bananas will be eaten along the way!

   (a) One way to successfully deliver at least some Bananas is to first send the Elephant to an initial stop 40 miles away with a full load. When he arrives, there will be 1100 pounds remaining. He leaves 700 pounds, and takes 400 pounds for the return to Baton Rouge. Now send the Elephant with a full load once again; he arrives at the stop with 1100 pounds, but can be replenished to a full load by adding another 400 pounds from the pile. Now he travels the remaining 110 miles, consuming 1100 pounds along the way, delivering **400** pounds. Is it possible to deliver more Bananas if a different initial stop is chosen?

   (b) Find the maximum amount of Bananas that can be delivered.

   Hint: The answer is greater than 500 pounds!

6. The vertices of a convex polygon with an even number of vertices are colored Red or Blue such that there are an equal number of vertices of both colors. Is it always possible to draw a set of straight lines such that (i) each line connects two vertices of opposite colors, and (ii) none of the lines intersect?

7. [**Gelca-Andreescu 31**] The vertices of a convex polygon are colored by at least three colors such that no two consecutive vertices have the same color. Prove that one can dissect the polygon into triangles by diagonals that do not cross and whose endpoints have different colors.

8. [Putnam **1962 B4**] A finite set of circles divides the plane into regions. Show that we can color the plane with two colors so that no two adjacent regions (with a common arc of non-zero length forming part of each region's boundary) have the same color.