- Virginia Tech Mathematics Contest. Sat., Oct. 26. **Sign-up deadline: Sep. 27**.
- Putnam Mathematical Competition. Sat., Dec. 7. **Sign-up deadline: Nov. 1**.

LSU Problem Solving Seminar - Fall 2019
Aug. 28: Introduction and Miscellany

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Warm Up

1. A team of 3 master Woodworkers can complete 100 feet of trim in a day, whereas a group of 5 Apprentices can only complete 60 feet of trim. How long will it take 6 Woodworkers and 8 Apprentices to finish 4440 feet of trim?

2. (a) Draw a picture that explains why the simple identity

$$ab + ka + \ell b = (a + k)(b + \ell) - k\ell$$

   is often known as "Completing the Rectangle".

   (b) Find all integer solutions to

$$xy + 3x + 5y = 38.$$

3. Let $f(x) = (x + 1)(x + 3)(x + 6)$.
   (a) Evaluate $f'(0)$.
   (b) Evaluate $f'''(0)$.

Main Problems

4. Let $f(x) := xe^{-x^2}$.
   (a) Calculate $f'(x)$.
   (b) Evaluate $f''(0)$.
   (c) Evaluate $f^{(7)}(0)$. Try to do this **without** explicitly writing down the general formula for the seventh derivative!

5. In this problem you will consider configurations of coins in the hexagonal (or "honeycomb") lattice. An example configuration is illustrated in Figure 1. In this figure there is an equilateral triangle whose vertices all have the same face (Heads).

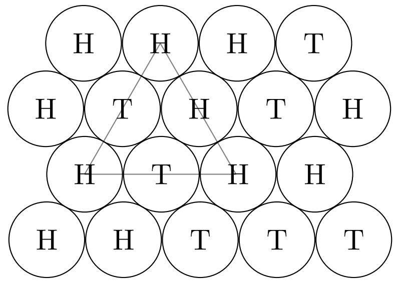

Figure 1: Coins in a hexagonal lattice, with an equilateral triangle of Heads.

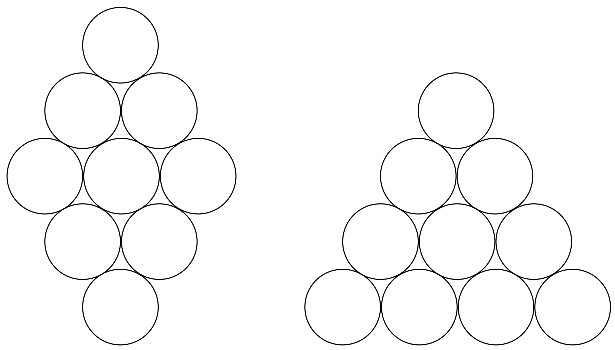

Figure 2: Coins in a diamond shape and pyramid shape.

(a) Find a configuration of coins in the "diamond" shape (see Figure 2) such that there is **no** equilateral triangle formed by coins with the same faces. How many such configurations are there?

(b) Is it possible to find a configuration of coins in a pyramid shape with no equilateral triangle of coins with the same faces?

6. (a) Show that the interval $[0, 1)$ can be decomposed into two disjoint sets A and B such that B is a *translate* of A, i.e. $A \cap B = \emptyset$ and $B = A + a$ for some real number a.

   (b) In fact, there is more than one way to decompose $[0, 1)$ in this manner – find another pair of disjoint subsets A and B.

7. [From **Gelca-Andreescu 10**] Is it possible to decompose $[0, 1]$ into two disjoint sets A and B whose union covers the whole interval, such that $B = A + a$ for some a?

   Hint: Consider the point 0 *– is it in* A *or* B*? What about other points near 0?*

8. [VTRMC **1981 # 7**] Let $A = \{a_0, a_1, \dots\}$ be a sequence of real numbers, and define the sequence $A' = \{a'_0, a'_1, \dots\}$ as follows for $n = 0, 1, \dots$:

$$a'_{2n} := a_n, \qquad a'_{2n+1} := a_n + 1.$$

   Suppose that $a_0 = 1$ and $A' = A$.

(a) Find a_1, a_2, a_3 and a_4.

(b) What is a_{1981}? What is a_{2019}?

(c) Give a simple algorithm for evaluating a_n in general.

9. (a) Find all integer solutions to
$$\frac{1}{a} + \frac{2}{b} = \frac{4}{71}.$$

(b) Show that there are no integer solutions to
$$\frac{1}{a} + \frac{1}{b} = \frac{3}{2017}.$$

Hint: 2017 is prime.

10. [Putnam **2018 A1**] Find all ordered pairs (a, b) of positive integers for which
$$\frac{1}{a} + \frac{1}{b} = \frac{3}{2018}.$$