Suppose that a right triangle has a hypotenuse of length h_0, vertical side of length 1, and horizontal side of length x_0. We would like to understand the behavior of the hypotenuse h as we change the length of the horizontal side to a different value x. The Pythagorean Theorem implies that as a function of x,

$$h(x) = \sqrt{1 + x^2}.$$

The linear approximation to this function about the value $x = x_0$ is

$$\begin{aligned} h(x) &\approx h(x_0) + h'(x_0) \cdot (x - x_0) \\ &= \sqrt{1 + x_0^2} + (1 + x_0^2)^{-1/2} \cdot x_0 \cdot (x - x_0), \end{aligned}$$

since

$$h'(x) = \frac{1}{2}(1 + x^2)^{-1/2} \cdot 2x.$$

Therefore if $x = x_0 + \Delta x$, then

$$\begin{aligned} \Delta h &= h - h_0 \\ &= \sqrt{1 + x_0^2} + (1 + x_0^2)^{-1/2} \cdot x_0 \cdot (x - x_0) - \sqrt{1 + x_0^2} \\ &= (1 + x_0^2)^{-1/2} \cdot x_0 \cdot \Delta x = \frac{x_0}{h_0} \cdot \Delta x. \end{aligned}$$

This can be rewritten in terms of percentage changes for both h and x as

$$\begin{aligned} \frac{\Delta h}{h_0} &= \frac{x_0}{h_0^2} \cdot \Delta x = \frac{x_0^2}{h_0^2} \cdot \frac{\Delta x}{x_0} \\ &= \cos^2 \alpha_0 \cdot \frac{\Delta x}{x_0}, \end{aligned}$$

where α_0 is the angle formed between the hypotenuse and horizontal leg in the initial triangle.

Finally, if $\alpha_0 = \pi/6$ and x increases by 2%, then

$$\frac{\Delta h}{h_0} = \frac{3}{4} \cdot .02 = .015,$$

so h increases by 1.5%.