Adaptive Tracking and Parameter Identification for Nonlinear Control Systems

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Holder of Roy Paul Daniels Professorship #3
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Typically we construct $u(t, Y)$ so that all trajectories of (2) for all possible choices of δ satisfy some control objective.

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Find γ_i's by building certain strict LFs for $\dot{Y} = \mathcal{G}(t, Y, 0)$.

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We transform V_1 into the desired strict LF.

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Theorem: We can construct $\mathcal{L} \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\sharp(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) \stackrel{\text{def}}{=} \mathcal{L}\big(V_1(t,\tilde{\xi},\tilde{\theta},\tilde{\psi})\big) + \sum_{i=1}^{s} \overline{\Omega}_i(t,\tilde{\xi},\widetilde{\theta},\widetilde{\psi}) \quad , \tag{13}$$

$$\begin{aligned} \text{where} \quad \overline{\Omega}_i(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) \quad &= \quad -\tilde{z}_i\lambda_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \\ &\quad + \tfrac{1}{T\overline{\psi}}\alpha_i^\top(\widetilde{\theta}_i,\widetilde{\psi}_i)\Omega_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \quad , \end{aligned} \tag{14}$$

$$\begin{aligned} \alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \quad &= \quad \begin{bmatrix} \tilde{\theta}_i\psi_i - \theta_i\tilde{\psi}_i \\ \widetilde{\psi}_i \end{bmatrix}, \quad \text{and} \\ \Omega_i(t) \quad &= \quad \textstyle\int_{t-T}^{t}\int_{m}^{t}\lambda_i^\top(s)\lambda_i(s)\mathrm{d}s\,\mathrm{d}m \quad , \end{aligned} \tag{15}$$

is a strict LF for the $Y=(\tilde{\xi},\tilde{\theta},\tilde{\psi})$ dynamics on $\mathcal{Y}$, so it is UGAS.

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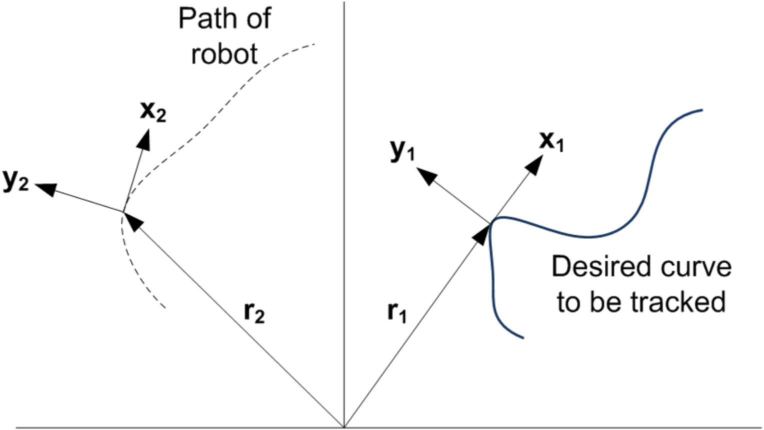

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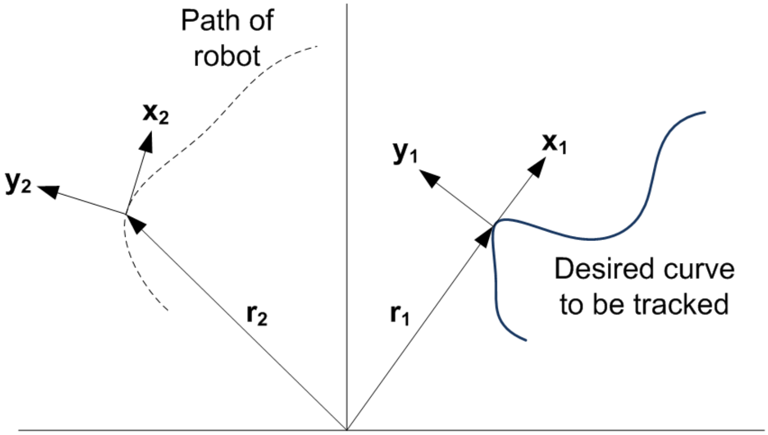

$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

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$$h(\rho) = \alpha \left\{ \rho + \frac{\rho_0^2}{\rho} - 2\rho_0 \right\}, \quad \rho_0 = \text{desired value for } \rho \tag{18}$$

$$V(\rho, \phi) = -\ln\big(\cos(\phi)\big) + h(\rho) \tag{19}$$

$$U(\rho, \phi) = -h'(\rho)\sin(\phi) + \frac{1}{\mu} \int_0^{V(\rho,\phi)} \Gamma_0(m)\mathrm{d}m \tag{20}$$

Robustly Forwardly Invariant Hexagons

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and $\delta : [0, \infty) \to [-\delta_{*i}, \delta_{*i}]$, on certain forward invariant sets H_i.

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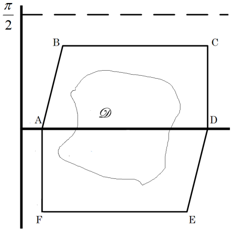

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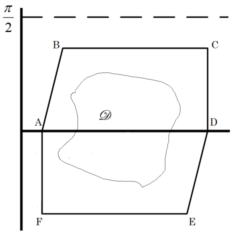

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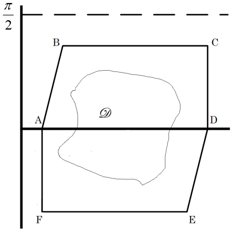

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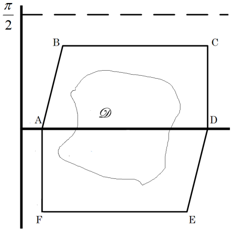

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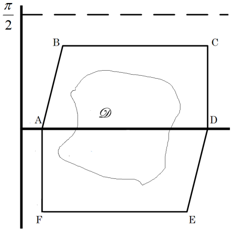

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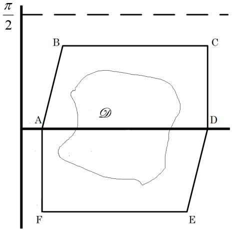

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$\Delta_{*i} = \min\{|h'(\rho)\cos(\phi)| : (\rho,\phi)^\top \in \mathrm{AB} \cup \mathrm{ED}\}$
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New Results (Mazenc, de Queiroz, M., '11)

We solved the tracking and parameter identification problem for

$$\left\{ \begin{array}{lcl} \dot{x} & = & f(\xi) \\ \dot{z}_i & = & g_i(\xi) + k_i(\xi)\theta_i + \psi_i u_i, \quad i = 1, 2, \ldots, s\,. \end{array} \right. \tag{5}$$

$\xi = (x, z) \in \mathbb{R}^{r+s}$. $(\theta, \psi) = (\theta_1, ..., \theta_s, \psi_1, \ldots, \psi_s) \in \mathbb{R}^{p_1+\ldots+p_s+s}$.

The C^2 T-periodic reference trajectory $\xi_R = (x_R, z_R)$ to be tracked is assumed to satisfy $\dot{x}_R(t) = f(\xi_R(t))$ $\forall t \geq 0$.

Main PE Assumption: positive definiteness of the matrices

$$\mathcal{M}_i \stackrel{\text{def}}{=} \int_0^T \lambda_i^\top(t)\lambda_i(t)\,\mathrm{d}t \in \mathbb{R}^{(p_i+1)\times(p_i+1)}, \tag{6}$$

where $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$ for $i = 1, 2, \ldots, s$.

Adaptive Robust Curve Tracking

$$\begin{cases} \dot{\rho} & = & -\sin(\phi) \\ \dot{\phi} & = & \frac{\kappa\cos(\phi)}{1+\kappa\rho} + K[u+\delta] \end{cases} \quad (22)$$

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$$\begin{cases} \dot{\tilde{q}}_1 & = & -\sin(\tilde{q}_2) \\ \dot{\tilde{q}}_2 & = & \frac{\kappa\cos(\tilde{q}_2)}{1+\kappa(\tilde{q}_1+\rho_0)} - \frac{K}{\tilde{K}+K}u_b + K\delta \\ \dot{\tilde{K}} & = & -(\tilde{K}+K-c_{\min})(c_{\max}-\tilde{K}-K)\frac{\partial U}{\partial \phi}\frac{u_b}{\tilde{K}+K} \end{cases} \tag{23}$$

for $(\tilde{q}_1, \tilde{q}_2, \tilde{K}) = (\rho-\rho_0, \phi, \hat{K}-K)$ on each set in a nested sequence of hexagonal regions that fill the state space.

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Georgia Tech Savannah Robotics Team (led by Fumin Zhang).

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(Loading Video...)

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- We aim for extensions that cover input delays and state constraints that ensure collision avoidance.