
DEPARTMENTAL COLLOQUIUM

3:40 PM, 4 May 2006, 284 Lockett Hall
Louisiana State University, Baton Rouge

Franco Rampazzo
Professor of Mathematical Analysis
Dipartimento di Matematica Pura ed Applicata
Università degli Studi di Padova, ITALY

Commutators of Flows of Nonsmooth Vector Fields

My talk will concern the problem of finding a nonsmooth analogue of the notion of Lie bracket—the *commutator*—of two vector fields. The results I am presenting are contained in joint works with H. Sussmann. In particular, by means of a notion of set-valued Lie bracket introduced in [1], in [2] we have extended some classical results valid for smooth vector fields to the case when the vector fields are just Lipschitz. For instance, we have proved that the flows of two Lipschitz vector fields *commute* for small times if and only if their set-valued Lie bracket vanishes everywhere (or, equivalently, if their classical Lie bracket vanishes almost everywhere). We have also extended the asymptotic formula that gives an estimate of the lack of commutativity of two vector fields in terms of their Lie bracket, and we have proved a *simultaneous flow box theorem* for commuting families of Lipschitz vector fields. Finally, I shall mention the question of higher order Lie brackets. In particular, the extension of the concept of set-valued Lie bracket to an order greater than two requires some care, and it cannot be merely treated by inductively applying the notion of degree-two bracket. However, we are proposing a concept of higher degree bracket which, in particular, allows us to generalize the classical Chow’s Theorem.

- [1] Rampazzo, F., and H.J. Sussmann, “Set-valued differentials and a nonsmooth version of Chow’s theorem,” in *Proceedings of the 40th IEEE Conference on Decision and Control (Orlando, FL, December 2001)*, Vol. 3, pp. 2613-2618.
- [2] Rampazzo, F., and H.J. Sussmann, “Commutators of flows of nonsmooth vector fields,” submitted for publication.

Professor Rampazzo’s visit to Louisiana State University is sponsored by the Louisiana Board of Regents grant “Enhancing Control Theory at LSU” (Primary Host: Peter Wolenski, wolenski@math.lsu.edu).
Campus Map: <http://www.lsu.edu/campus/locations/LCKT.html>.

CONTROL THEORY SEMINAR

3:40 PM, 11 May 2006, 381 Lockett Hall
Louisiana State University, Baton Rouge

Franco Rampazzo
Professor of Mathematical Analysis
Dipartimento di Matematica Pura ed Applicata
Università degli Studi di Padova, ITALY

Moving Constraints as Controls in Classical Mechanics

In most applications of control theory to mechanics the *control* is identified with a force, or with a torque. However, in some concrete situations, the forces are in fact unknown, whereas what one is actually controlling is the *position of part of the system*. More precisely, if the state space consists of the product $Q \times C$ of two manifolds Q and C , one can regard Q as the actual (reduced) state space by identifying C with a set of controls. As an example, one can think of a mathematical pendulum whose pivot is constrained on a vertical line. In this case $Q = S^1$ and $C = \mathbf{R}$. (The title of the talk refers to the fact that a control function $\mathbf{c}(\cdot)$ defined on a time-interval I can be considered as a time dependent (i.e., *moving*) state-constraint acting on the original state space $Q \times C$.)

To begin with, we will illustrate some remarkable geometric aspects, which involve, in particular, the metric induced by the kinetic energy on the manifold $Q \times C$ and its relation with the foliation $\{Q \times \{\mathbf{c}\} \mid \mathbf{c} \in C\}$.

Secondly, we will address the question of the closure of the set of solutions for unbounded control systems, and we will see how this issue is connected with our mechanical problems.

Finally, we will show how some well-known mechanical questions—including the vibrational stabilization of the so-called *inverted pendulum*—can actually be regarded as instances of problems involving moving constraints as controls.

Professor Rampazzo's visit to Louisiana State University is sponsored by the Louisiana Board of Regents grant "Enhancing Control Theory at LSU" (Primary Host: Peter Wolenski, wolenski@math.lsu.edu). Campus Map: <http://www.lsu.edu/campus/locations/LCKT.html>.
